**Exercise 1.** ***Equilibrium in electrostatic field and behaviour inside a cavity of a conductor***

1. Can a point charge be in a stable mechanical equilibrium in the electric field of other charges?

   (a) Assume first that $P_0$ is a point of a stable equilibrium in an electrostatic field. What is the force acting on a charge in equilibrium at the point $P_0$? How does the electric field act around $P_0$?

   (b) Consider now a Gaussian surface at a small distance around $P_0$. Apply Gauss' law and conclude that there must be a charge inside the considered volume if we assume $P_0$ to be a stable equilibrium point.

   (c) What can you conclude about the equilibrium in the electrostatic fields?

2. Consider a conductor with a cavity. There is no electric field in the metal, but what about in the cavity?

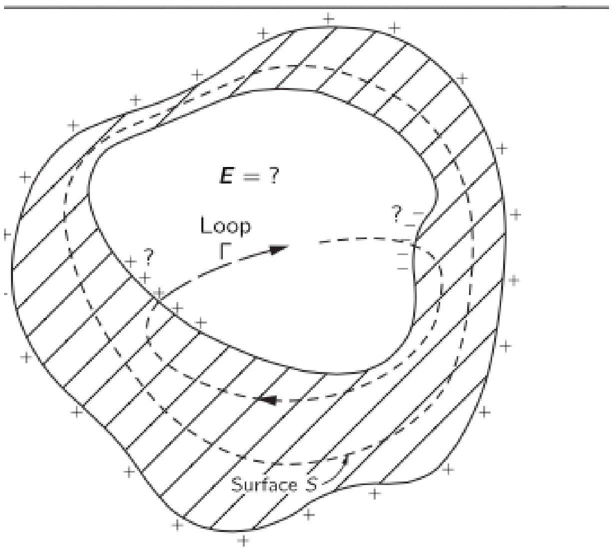


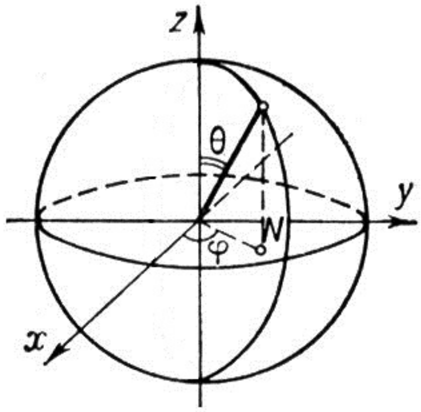

Figure 1: *Cavity*

   (a) Let us consider a cavity of a conductor (for any shape) like the one given in Fig.1. Consider a Gaussian surface, *e.g.* $S$ in the figure, that encloses the cavity but stays everywhere in the conducting material. Could there be a positive surface charge on one part and a negative one somewhere else, as indicated in the figure?

   (b) Now imagine a loop that crosses the cavity along a line of force from some positive to some negative charge, and returns to its starting point via the conductor (see Fig.1). What can you conclude by applying Stoke's theorem?

Hint: This exercise is based on the Feynman lectures Vol. 2, Ch.05-4. You can find the answer in the link below, but it will be good to test your ability to answer yourselves first.
*www.feynmanlectures.caltech.edu*

**Exercise 2.** ***Electric field of a sphere***




Consider a spherical shell of radius $R_0$, charged with an electric charge $Q$ which is uniformly distributed.

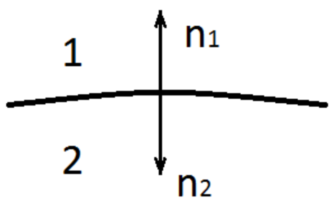
a) Write down the charge density $\rho(\mathbf{x})$ of the shell.

b) Using the symmetry of the system and Gauss' law, find the electric field $\mathbf{E}$ inside and outside of the shell.

c) Consider an element of arbitrary surface with charge density $\rho$ shown in the figure. Show that the electric field due to the charges external to this surface element can be calculated as

$$\mathbf{E}_{ext} = \frac{\mathbf{E_1} + \mathbf{E_2}}{2}, \tag{1}$$

where $\mathbf{E_1}$ and $\mathbf{E_2}$ correspond to the electric fields in regions 1 and 2.




d) Now, consider the sphere of parts a) and b) to be split into two hemispheres by the plane $z = 0$. Using the results from all the previous parts, calculate the force with which the two hemispheres repel each other.

**Exercise 3.** ***Energy of a charge distribution***

Find the energy stored in two different configurations:

1. a charged spherical conductor
2. a uniformly charged solid sphere

both with radius $R$ and charge $Q$. Explain the energy difference between the two cases.

### Exercise 4. *The Electrostatic Potential of NaCl*

In this exercise we will calculate the energy per molecule required to separate the ionic crystal NaCl (salt) into its ions.

The NaCl crystal consists of negative and positive ions whose structure we know from x-ray diffraction. It is a cubic lattice and the cross section of the crystal is depicted in Fig.2. We assume that the ions $Na^+$ and $Cl^-$ are of the same size. The energy of this lattice originates from the electrical interaction of the atoms, a repulsion which becomes important if ions are compressed too close together, and a kinetic energy due to vibrations. In this problem we will ignore the repulsive force and the lattice vibrations and approximate the required energy to pull apart the NaCl ions as the electrostatic potential between the ions.

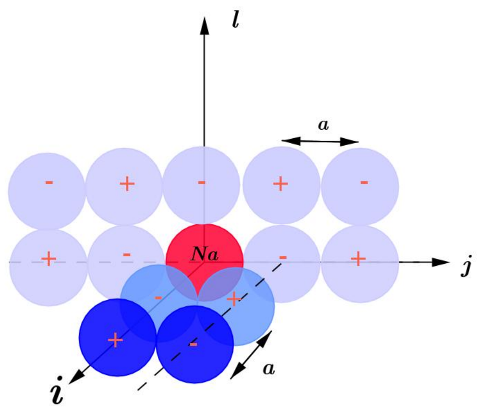



Figure 2: *NaCl*

1. Consider only one ion assigned as $Na^+$ in the figure. Write down the energy due to the electrostatic interaction of this ion and the neighbours in the horizontal line along the $i-$axis. The distance between the $Na^+$ ion assigned in the Figure and the nearest $Cl^-$ is $a = 2.81\,10^{-8}$ cm. Evaluate the sum analytically.
2. Now consider the nearest next line and write down the sum for the electrostatic potential. Note that there are four such lines surrounding the chosen ion.

3. Write down an expression taking into account the rest of the lines. Evaluate your result numerically (in the language of your choice). Now compare your result to the experimentally measured result of 7.92 eV per molecule. Are they of the same order of magnitude? How would you interpret the small discrepancy?