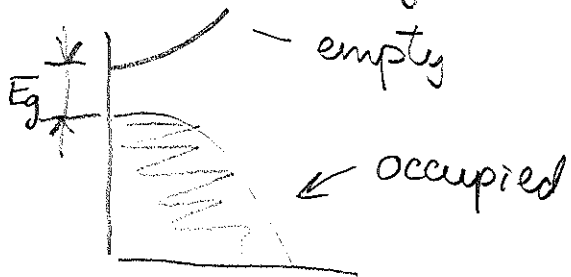


Semiconductors are band insulators with relatively low energy gap E_g

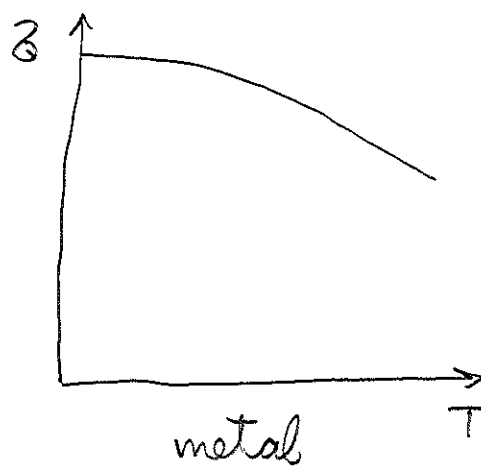
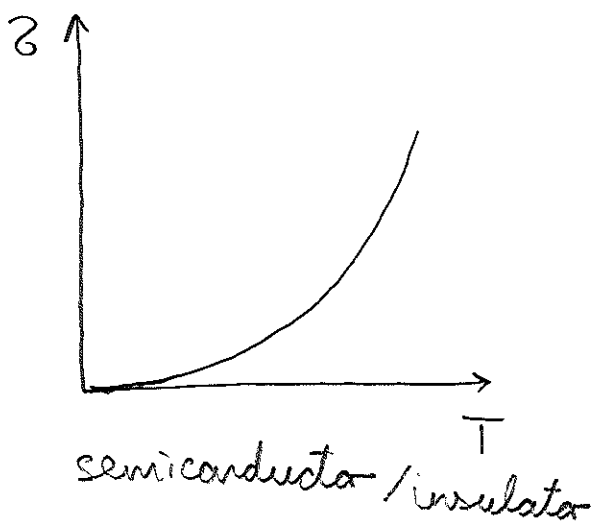


Density of thermally excited carriers is $\propto e^{-E_g/2T}$

For typical semiconductors $E_g < 2\text{ eV}$

The distinction between an insulators and a semiconductors is not a sharp one.

Conductivity



The conductivity may be written in the Drude form: ¹²

$$\beta = \frac{ne^2\tau}{m}$$

Quick and dirty derivation:

Typical carrier velocity is given by the Newton's law

$$m \frac{v}{\tau} = eE \Rightarrow v = \frac{eE\tau}{m}, \tau - \text{scattering time}$$

$$\text{Current } j = nev = \frac{ne^2\tau}{m} E \Rightarrow$$

$$\beta = \frac{ne^2\tau}{m}$$

Here n is density of mobile carriers, m is their mass. In metals n is temperature independent and scattering ^{rate} is increased with increasing temperature. Thus temperature dependence of β in metals is given by $\tau(T)$. In semiconductors and insulators there are no mobile carriers at $T=0 \Rightarrow \beta(T=0) = 0$. At finite temperatures carriers are induced by thermal activation above the gap E_g between the valence and conduction bands.

$$n = n_0 \left(\frac{T}{T_0} \right)^{3/2} e^{-\frac{E_g}{2T}}$$

$$T_0 \approx 300 \text{ K}, \quad n_0 \approx 10^{20} \text{ cm}^{-3}$$

For insulators like diamond $E_g = 5.5 \text{ eV}$ and at room temperature $n \sim 10^{-27} \text{ cm}^{-3}$.

Reducing gap to $E_g \sim 0.5 - 1 \text{ eV}$ we can achieve measurable values of carrier densities $n \sim 10^5 - 10^{10} \text{ cm}^{-3}$ (which is still much less than metallic one $n \sim 10^{23} \text{ cm}^{-3}$).

The conductivity of a semiconductor is not a good measure of scattering rate, as it is in metals.

To describe collisions one defines mobility μ of a carrier as $v = \mu E$

$$\text{Then current } j = nev \Rightarrow \sigma = ne\mu$$

In this way one can separate temperature dependence of conductivity mainly given by $n(T)$ and purity of the sample that is characterized by μ

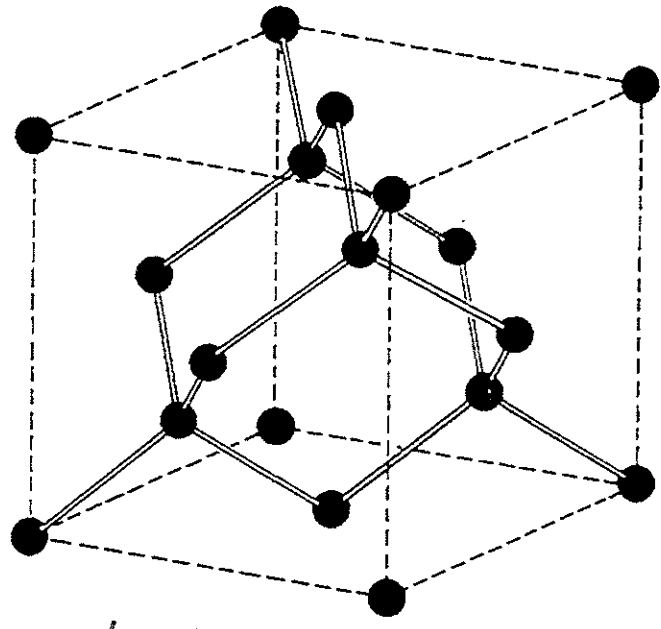
Crystal and Band structure

(4)

The most important semiconductor for technological applications is silicon (Si). It belongs to the group IV of

the periodic table (as C, Ge, Sn)

All four elements have diamond structure



which is face-centered cubic lattice (FCC) with a unit cell containing two atoms located at

$(0,0,0)$ and $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. The outer shell of the group IV elements has 2s and 2p electrons

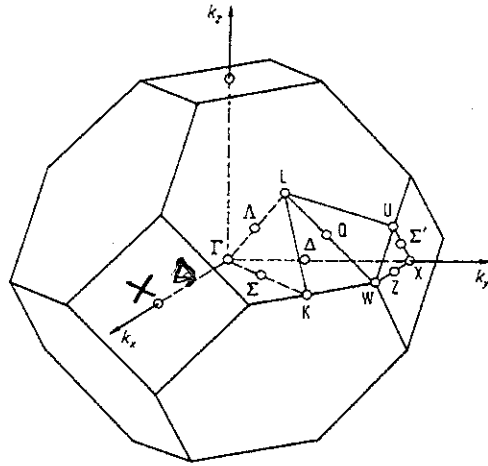
These four valent electrons are hybridized and form covalent bonds of the crystal.

Another typical semiconductor GaAs is made from the group III and V elements. It has the same structure as Si with two sublattices shifted by $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$

occupied by atoms of Ga and As respectively.

Such structure is called zinc blende structure

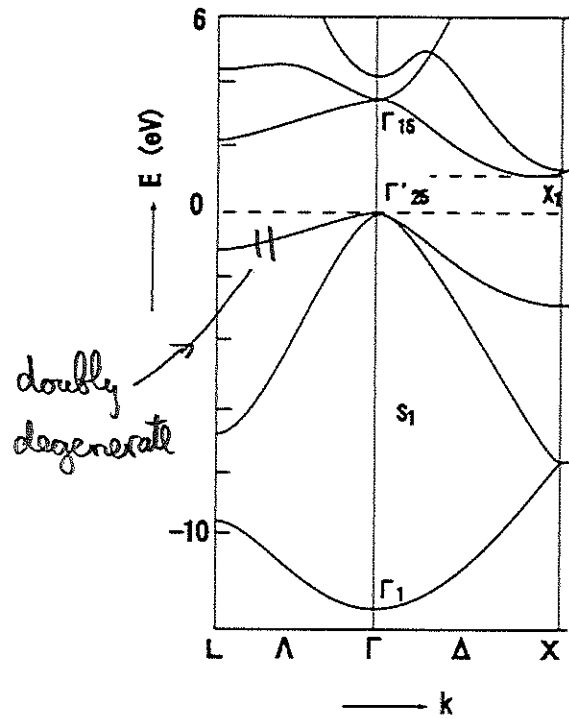
The Brillouin zone of the FCC lattice is



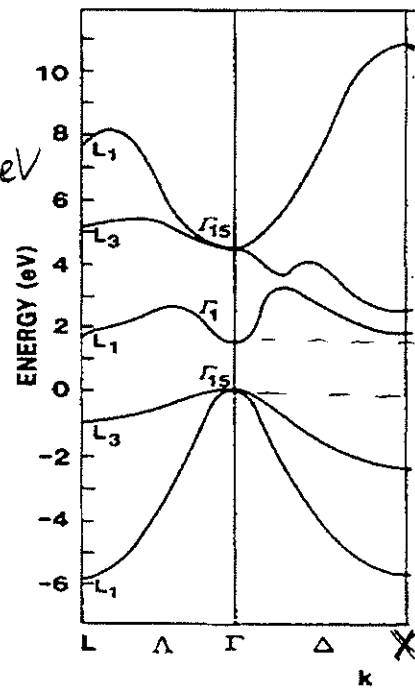
The band structure along the $L - \Gamma - X$ line is presented below. At Γ point there are singly degenerate and triply degenerate levels

Si

GaAs



$E_g = 1.12\text{eV}$

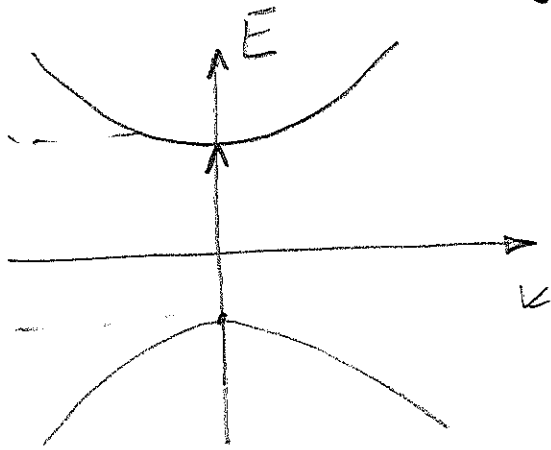


$E_g = 1.43\text{eV}$

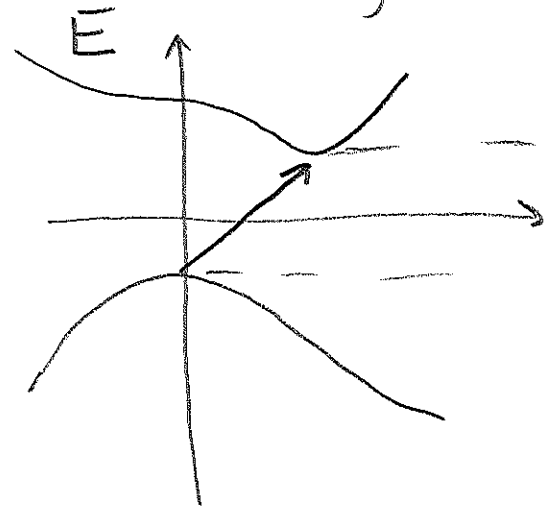
Two atoms per unit cell provide 8 electrons that fill 4 lowest bands.

For silicon the top of the valence band is at $k=0$ (6)
but the bottom of the conduction band is at $k_0 \neq 0$
We call this indirect gap.

For GaAs has direct gap where
maximum of the valence band and minimum
of conduction bands have the same
pseudo momentum ($k=0$ in this case)



direct energy gap



indirect energy gap.

The difference between direct and indirect gaps become important when one considers optical properties of semiconductors (light absorption)

Elementary excitations

Consider simple two band model with the Hamiltonian

$$H = \sum_{\mathbf{k}, s} \epsilon_{v, \mathbf{k}} C_{v, \mathbf{k} s}^{\dagger} C_{v, \mathbf{k} s} + \sum_{\mathbf{k}, s} \epsilon_{c, \mathbf{k}} C_{c, \mathbf{k} s}^{\dagger} C_{c, \mathbf{k} s},$$

where $\epsilon_{v, \mathbf{k}}$ and $\epsilon_{c, \mathbf{k}}$ are the band energies of the valence and conduction band respectively. $C_{v, \mathbf{k} s}^{\dagger}$ ($C_{c, \mathbf{k} s}^{\dagger}$) creates an electron in the valence (conduction band). The ground state energy

$$\langle 0 | E_0 = 2 \sum_{\mathbf{k}} \epsilon_{v, \mathbf{k}}.$$

To find the chemical potential and the density of excited electrons we assume parabolic spectrum near the top and ^{the} bottom of the valence and conduction bands

$$\epsilon_{v, \mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m_v} \quad \epsilon_{c, \mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m_c} + E_g$$

The overall electron density is n

$$n = \sum_{\mathbf{k} \in \text{BZ}} \sum_s 1 = 2 \int_{\text{BZ}} \frac{d^3 \mathbf{k}}{(2\pi)^3}$$

The density of electrons in the conduction band is (8)

$$n_c = 2 \int_{BZ} \frac{d^3 k}{(2\pi)^3} n_F(\epsilon_{c,k}) = \int_{BZ} \frac{d^3 k}{4\pi^3} \frac{1}{e^{(\epsilon_{c,k} - \mu)/T} + 1} \approx$$
$$\approx \int_{BZ} \frac{d^3 k}{4\pi^3} e^{-\left(\frac{\epsilon_{c,k} - \mu}{T}\right)} = e^{\frac{(\mu - E_g)}{T}} \frac{(2\pi m_c T)^{3/2}}{4\pi^3}$$

Here we assumed $E_g - \mu \gg T$

Analogously in the valence band we have

$$n - n_v = 2 \int_{BZ} \frac{d^3 k}{(2\pi)^3} (1 - n_F(\epsilon_{v,k})) = \int_{BZ} \frac{d^3 k}{4\pi^3} \frac{1}{\left(e^{-\frac{(\epsilon_{v,k} - \mu)}{T}} + 1\right)}$$
$$= \int_{BZ} \frac{d^3 k}{4\pi^3} e^{\frac{(\epsilon_{v,k} - \mu)}{T}} = e^{-\frac{\mu}{T}} \frac{(2\pi m_v T)^{3/2}}{4\pi^3}$$

But the total number of electrons is conserved, thus $n_c + n_v = n$ and equating

$$e^{\frac{\mu - E_g}{T}} m_c^{3/2} = e^{-\frac{\mu}{T}} m_v^{3/2} \Rightarrow$$

$$\mu = \frac{E_g}{2} + \frac{3}{4} T \ln\left(\frac{m_v}{m_c}\right)$$

At $T=0$ chemical potential is exactly in the center of the band gap.

Effective masses

(9)

In general near the top (bottom) of the band the effective mass is anisotropic.

$$E_c(\mathbf{k}) = \hbar^2 \left(\frac{k_1^2}{2m_1} + \frac{k_2^2}{2m_2} + \frac{k_3^2}{2m_3} \right) + E_g$$

For GaAs the top and the bottom of the band are at $\mathbf{k}=0$ where due to cubic symmetry mass is isotropic $m_1 = m_2 = m_3$

The same is true for the valence band of silicon where two degenerate bands meet with $M = 0.49 m_e$ and $m = 0.16 m_e$, m_e - free electron mass. For the conduction band that has minimum at the X point symmetry is lowered and one has $m_L = 1.0 m_e$ and $m_T \approx 0.2 m_e$ (longitudinal and transverse masses)

Cyclotron Resonance

In the presence of the DC magnetic field electron (hole) motion is given by

$$m \frac{d\vec{v}}{dt} = \mp \frac{e}{c} \vec{v} \times \vec{B}$$

Then we have precession with the cyclotron frequency

$$\omega_c = \frac{eB}{mc}$$

Applying AC electric field orthogonal to \vec{B} we get resonance absorption at

$$\omega = \frac{eB}{mc} = \omega_c$$

That's how one measures effective mass.

For anisotropic mass tensor one measures

$$m^* = \frac{\det(M_{\pm B})}{m_{zz}}$$

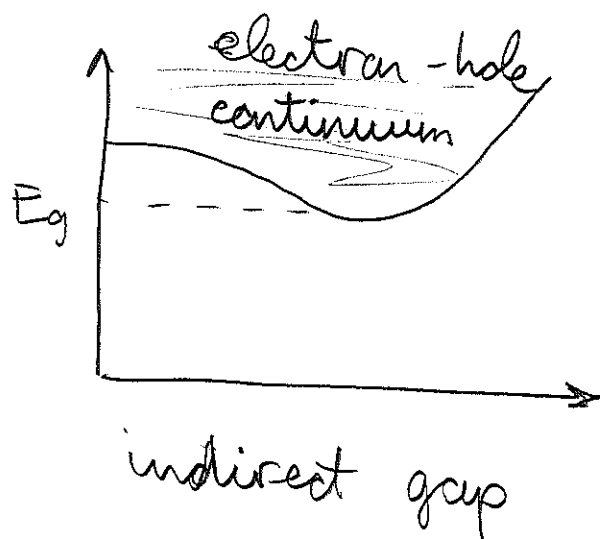
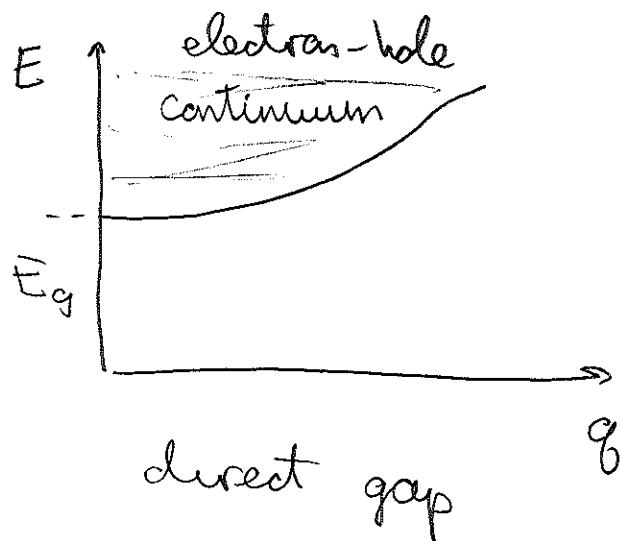
where m_{zz} is the mass

component along the field. To measure cyclotron resonance one needs $\omega_c \tau \gg 1$.

Electron hole excitations

A simple excitation in semiconductors consists of removing an electron (i.e., creating a hole) from the valence band with momentum k and spin S and inserting it into the conduction band with momentum $k+q$, and spin S'

$$|k+q, S'; k, S\rangle = C_{C, k+q, S'}^+ C_{V, k, S} |\Psi_0\rangle$$



The electron-hole excitations of a semiconductor are similar to electron-positron pairs in the Dirac theory

Excitons

Electrons and holes have opposite charges \Rightarrow they attract each other. They may form hydrogen like bound state with the energy

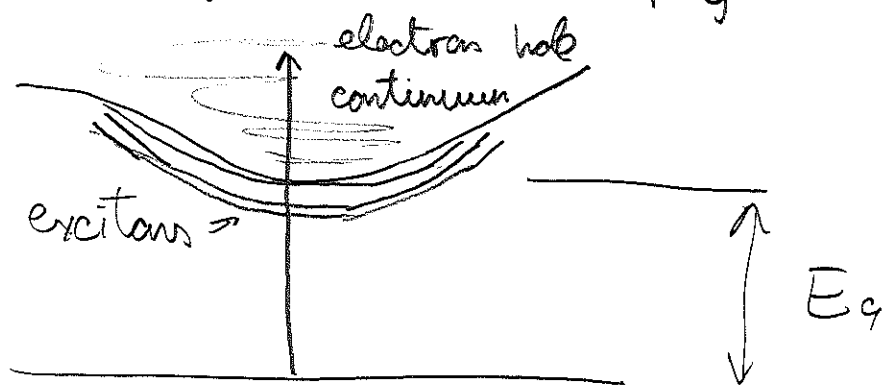
$$E(k) = E_g - \frac{\mu_{ex} e^4}{2 \epsilon^2 \hbar^2} + \frac{\hbar^2 k^2}{2 M_{ex}}$$

Here $\mu_{ex} = \frac{m_e m_h}{m_e + m_h}$ - reduced mass,

$M_{ex} \geq m_e + m_h$ - total mass,

ϵ - dielectric constant.

Typically $\mu_{ex} \approx 0.1 m_e$, $\epsilon \sim 10$ and exciton energy is $\sim 10^{-3} R_y$



Optical properties

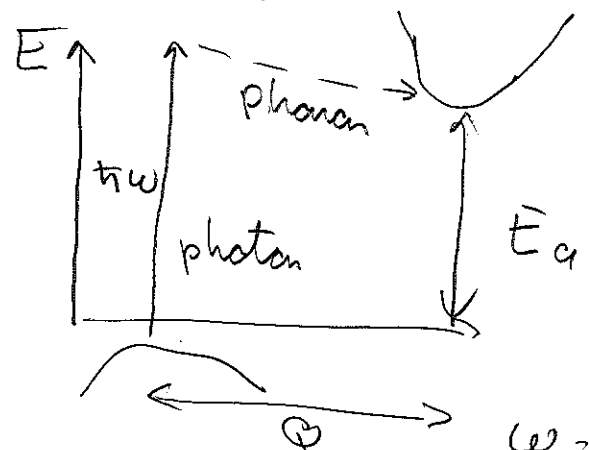
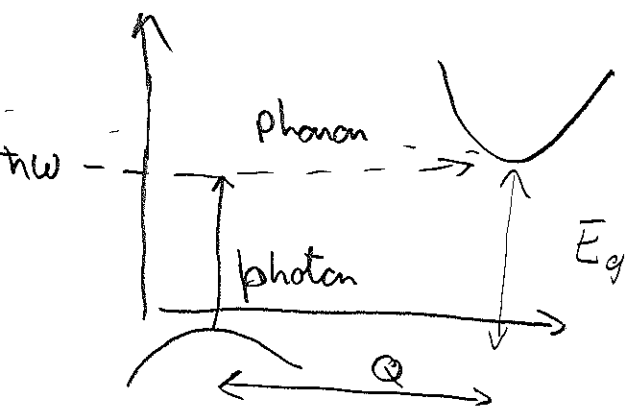
(13)

The conductivity of a semiconductor can be enhanced by shining light on a material. This is due to exciting of electron-hole pairs by absorption of photons.

The photon energy $\hbar\omega = ck \approx E_g \Rightarrow$
momentum $k \approx \frac{E_g}{c} \ll \frac{2\pi}{a}$

Thus pure electromagnetic excitations lead only to direct excitations with absorption rate $\Gamma_{abs}(\omega) \propto (\hbar\omega - E_g)^{1/2}$ for the dipole allowed transitions.

For semiconductors with indirect energy gap one needs either absorb or emit phonons



Then the transition rate is reduced

$$\Gamma_{abs} \propto C_+ (\hbar\omega + \hbar\omega_Q - E_g)^2 + C_- (\hbar\omega - \hbar\omega_Q - E_g)^2$$

$$\omega_Q = c_s(Q)$$