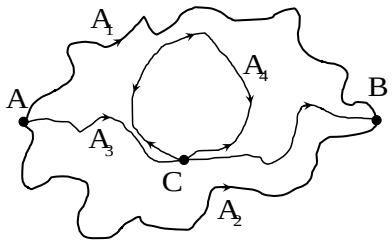
Lecture 22

Weak Localization. Variable range hopping

Quantum corrections to the conductivity

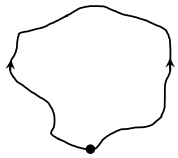
Qualitative picture (for experimentalists!)
Let us start from the Drude formula $\sigma = ne^2\tau/m$. Consider a good conductor with the electron wavelength $\lambda \ll l$ and low temperature $T \to 0$. The probability for electron to go from point A to point B is given by the square of modulus of the sum of all amplitudes:



$$W = \left|\sum_i A_i\right|^2 = \sum_i |A_i|^2 + \sum_{i\neq j} A_i A_j^*. \quad (1)$$

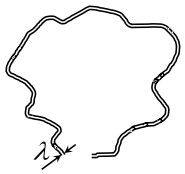
The first term in the Eq. (1) is the sum of the probabilities and corresponds to the classical Boltzmann approach. The last one describes the interference between the different trajectories. For most paths interference is not important since they have different length and acquire random phases $\int_A^B pdl/\hbar$. There are, however, special self-intersecting paths like A_4 from the point C.

For returning to the same point there are always two trajectories corresponding to the opposite directions of going around the loop.



In the presence of time reversal symmetry these two amplitudes are coherent and $W = |A_1|^2 + |A_2|^2 + 2A_1A_2^* = 4|A_1|^2$, that is twice more than the classical result of adding two probabilities. As a result of interference the probability of return to the origin is enhanced. Then the probability to go to another point is reduced and one gets decrease of conductivity.
To estimate correction to the conductivity we should sum over all possible self-intersections of the classical paths. For classical point the thickness of the path is zero and self-intersection is not possible in 3d case. For electron one should consider its trajectory as a tube of radius $\lambda \sim 1/k_F$. To return to the origin within the time dt means to be inside the volume $dV \sim vdt\lambda^{d-1}$,

v is electron velocity. During the time t particle can diffuse to the volume $V_D \sim (Dt)^{d/2}$, $D = v^2\tau/d$ - diffusion coefficient. The probability of self-crossing is $\int dV/V_D$,



$$\frac{\delta\sigma}{\sigma} = -\int_{\tau}^{\tau_\varphi} \frac{vdt\lambda^{d-1}}{(Dt)^{d/2}} \tag{2}$$

Here τ is the transport time and τ_φ is the time during which phase coherence is preserved. Integrating Eq.(2) gives

$$\delta\sigma \sim -\frac{e^2}{\hbar}\begin{cases} L_\varphi & d=1 \\ \ln\frac{L_\varphi}{L} & d=2 \quad L_\varphi = \sqrt{D\tau_\varphi} \\ \text{const} - \frac{1}{L_\varphi} & d=3 \end{cases} \tag{3}$$

Two or one dimensions mean that transverse sizes are smaller than L_φ. For high frequency $\omega\tau_\varphi \gg 1$, L_φ should be replaced by $L_\omega = \sqrt{D/\omega}$. The corrections (3) although small in λ/l govern the frequency and temperature dependence of the conductivity (since $\tau_\varphi \sim T^{-p}$). Even in 3d $\delta\sigma \sim -\sqrt{\omega}$. In coherent conductor $L_\varphi \to \infty$. Divergence of these corrections in 1 and 2d cases gives good arguments for the scaling theory of localization.

Physical meaning of τ_φ
It is the time during which the wave function retains its coherence. As an example consider some kind of inelastic scattering with typical energy transfer ω_{in} and mean time between collisions τ_{in}. Change in energy during the time t is $\Delta\varepsilon(t) \sim \omega_{\text{in}}\sqrt{t/\tau_{\text{in}}}$. Change in phase $\hbar\Delta\varphi(t) \sim \Delta\varepsilon(t)t \sim \omega t\sqrt{t/\tau_{\text{in}}}$. τ_φ is determined from the condition $\Delta\varphi(\tau_\varphi) \simeq 1$, thus $\tau_\varphi \sim (\hbar^2\tau_{\text{in}}/\omega_{\text{in}}^2)^{1/3}$. Energy relaxation time is determined from $\Delta\varepsilon(\tau_\varepsilon) \simeq \varepsilon$ then $\tau_\varepsilon \sim \tau_{\text{in}}(\varepsilon/\omega_{\text{in}})^2$. For $\varepsilon \gg \omega_{\text{in}}$, $\tau_\varepsilon \gg \tau_\varphi$. For $\varepsilon < \omega_{\text{in}}$ and $\omega_{\text{in}} > \hbar/\tau_{\text{in}}$ $\tau_\varphi \sim \tau_\varepsilon \sim \tau_{\text{in}}$. In general τ_φ is the shortest inelastic relaxation time.

Everything that destroys time reversal symmetry - reduces quantum corrections (e.g. magnetic field or spin orbital scattering). Consider effect of magnetic field. Traveling along the closed path electron's wave function acquires an additional phase factor,

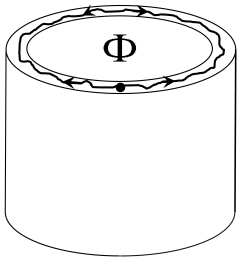
$$\Psi \to \Psi \exp\left(\frac{ie}{\hbar c}\int \mathbf{A}\mathbf{dl}\right) = \Psi \exp\left(\pm i\pi\frac{BS}{\Phi_\circ}\right),$$

where $\Phi_\circ = \pi\hbar c/e$ is superconducting flux quantum. The typical area S of

the loop is $\sim Dt$, thus one should cut off the interference correction on τ_B given by $D\tau_B B \sim \Phi_\circ$, or $\tau_B \sim \Phi_0/DB$. Replacing τ_ϕ by τ_B we obtain the field dependent corrections to conductivity

$$\sigma(H) - \sigma(0) \sim \frac{e^2}{\hbar} \begin{cases} \ln(eBD\tau_\phi/\hbar c) & d = 2 \\ \left(\frac{eB}{\hbar c}\right)^{1/2} & d = 3. \end{cases} \tag{4}$$

This leads to the negative magneto-resistance. In 3d it is independent on the angle between I and B. In 2d it is strongly anisotropic, determined by the orthogonal component of the field. Another consequence of the interference effect are oscillation of the resistivity of the thin hollow cylinder in parallel magnetic field. The period of these oscillations is the superconducting flux quantum. $\Phi_\circ = hc/2e$.



This is due to the fact that the phase difference between the waves traveling clockwise and anticlockwise is $\Delta\varphi = 2\pi\Phi/\Phi_\circ$.

Predicted by B. L. Altshuler, A. G. Aronov and B. Z. Spivak (1981), measured by D. Yu. Sharvin and Yu. V. Sharvin (1982).

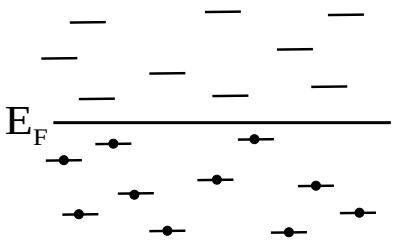
Variable range hopping

N. F. Mott (1968). Let us consider conductivity in the localized regime. At $T = 0$ the states below the Fermi level are localized and $\sigma(T = 0) = 0$. At $T > 0$ conductivity is non zero due to thermal activation. For activation to the mobility edge we obtain

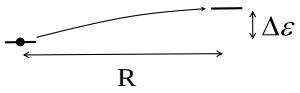
$$\sigma \propto \exp[-(E_m - E_F)/T].$$

However if we are not too close to mobility edge conductivity can appear due to thermally activated tunneling from one localized state to another.
Consider the system with the density of states $N(E)$. The typical level spacing in the volume R^d is

$$\Delta\varepsilon \sim 1/N(E_F)R^d. \tag{5}$$



Thus within the distance R the nearest free state is $\sim \Delta\varepsilon$ higher in energy.



In order to go to this state electron has to acquire energy $\Delta\varepsilon$ (from phonons) and tunnel over distance R. Then the probability is

$$P \propto \exp\left[-\frac{\Delta\varepsilon}{T} - \frac{2R}{L_c}\right],$$

where L_c is localization length. Substituting $\Delta\varepsilon(R)$ we obtain

$$P \propto \exp\left[-\frac{1}{TN(E_F)R^d} - \frac{2R}{L_c}\right].$$

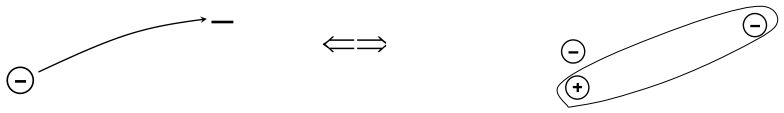
We should optimize this expression with respect to the hopping distance R. For large R the second term is important and probability of tunneling is too small. For small R the activation energy is too high. Optimal hopping distance depends on temperature, - variable range hopping,

$$R_h \sim \left[\frac{L_c}{TN(E_F)}\right]^{1/(d+1)},$$

and conductivity is

$$\sigma \propto \exp\left[-(T_0/T)^{1/(d+1)}\right], \qquad T_0 \sim \frac{1}{N(E_F)L_c^d}.$$

In 3d $\sigma \propto \exp[-(T_0/T^{1/4})]$ (Mott's law). In the Mott's picture Coulomb interaction is neglected. If the Coulomb energy $e^2/\varepsilon R$ (ε - dielectric constant) for creating the electron hole pair



becomes larger than $\Delta\varepsilon$ from Eq.(4) we should replace $\Delta\varepsilon$ by $e^2/\varepsilon R$ and

$$P \propto \exp\left[-\frac{e^2}{\varepsilon RT} - \frac{2R}{L_c}\right].$$

Optimizing for R we get

$$R \propto T^{-1/2} \qquad \text{and} \qquad \sigma \propto \exp\left[-(T_1/T)^{1/2}\right]$$

B. I. Shklovskii and A. I. Efros (1975).