ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Solid State Theory
Exercise 1

FS 15
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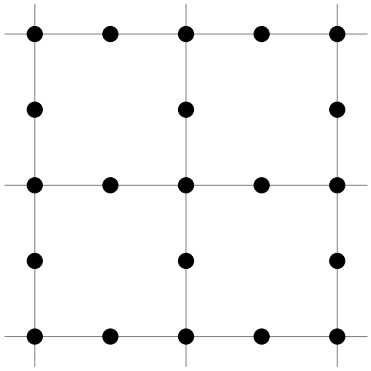
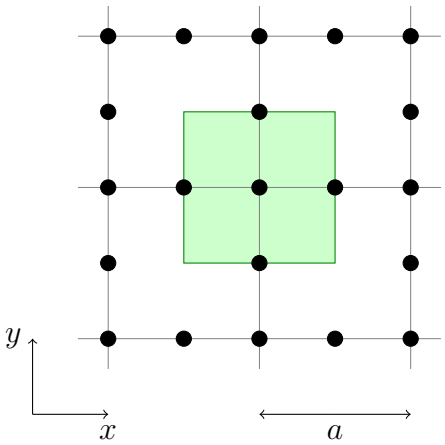
Problem 1.1 Side-centered square lattice

Consider a side centred square lattice.



1. To what type of Bravais lattice does it belong?
2. Draw the primitive cell. What is the basis?

Solution



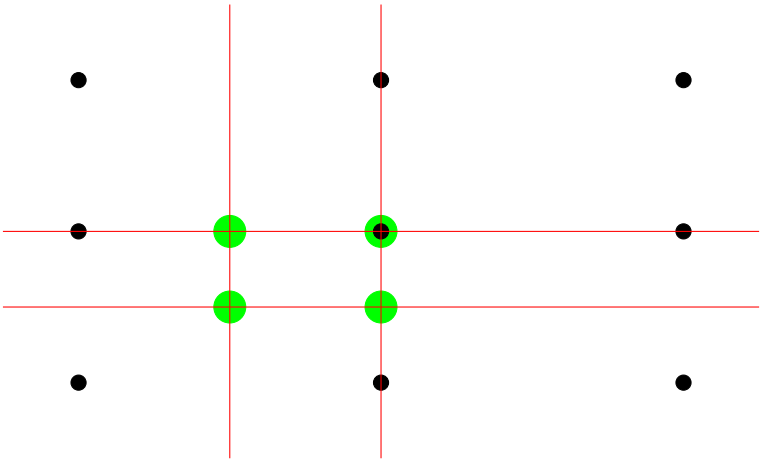
The lattice belongs to the square lattice type. The green shaded area is the Wigner-Seitz construction for the primitive cell. The basis is made up of three points located at $(0,0)$, $\left(\frac{a}{2},0\right)$ and $\left(0,\frac{a}{2}\right)$.
Tip: To make sure you have the right basis check that the total number of points in the primitive cell corresponds to the number of atoms in the basis and that no basis points are related by a lattice vector.

Problem 1.2 Centered rectangular lattice

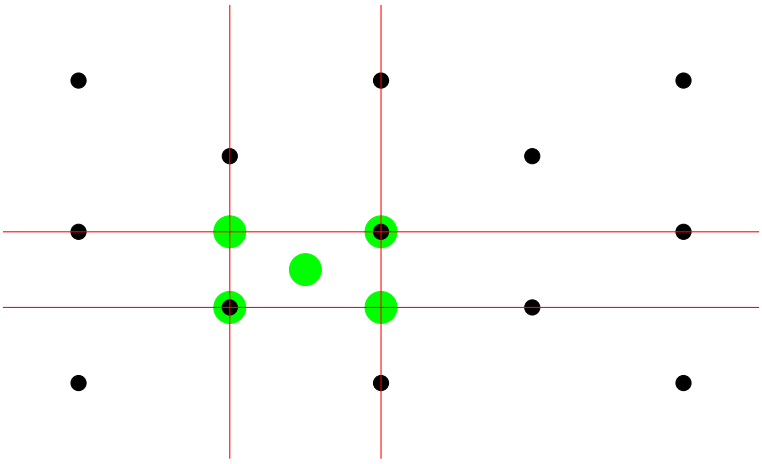
What is the difference in symmetry between the rectangular and the centered rectangular lattice? *Hint:* Check all possible symmetry operations of these lattices.

Solution

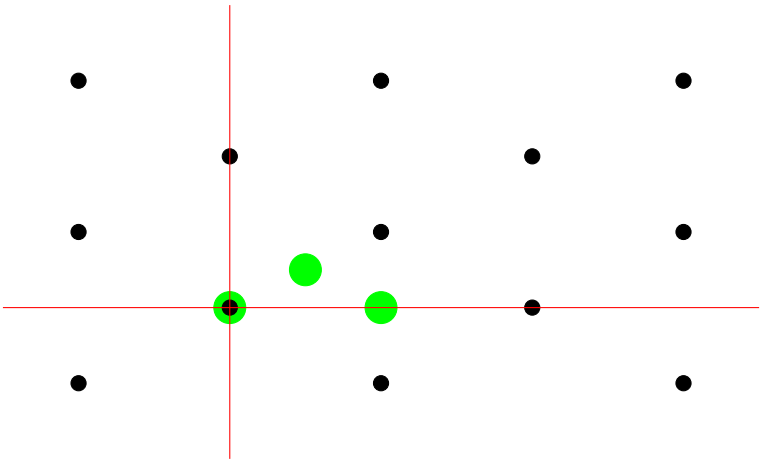
First we draw a rectangular lattice and all its inequivalent symmetries.



Here the red lines are the mirror planes and the green points are 180° rotations. We then do the same thing with the centred rectangular lattice using the same unit cell (not the primitive one).



It is very easy to see the additional point with a 180° rotation at the centre of the four "old" rotations. However the diagonal pairs of 180° rotations are equivalent through translations by rhombic primitive lattice vectors and the same goes for the pairs of mirror planes. The independent symmetry operations are shown here::



The number of independent symmetries is lower in the rhombic lattice.