Theoretical Physics, Problem Set 6.

FS15 *Hand in: 1.04.15*

1. Green's function in $\mathbb{R}^{2+1}$

i) Find the Green's function $D_2(\underline{x}, t)$ for the initial value problem

$$\Box u(\underline{x}, t) = 0, \qquad (\underline{x}, t) \in \mathbb{R}^{2+1},$$
$$u(\underline{x}, 0),\ \frac{\partial u}{\partial t}(\underline{x}, 0) \qquad \text{given}$$

in dimension 2.

Hint: Extend the problem to a 3-dimensional one, with $(\underline{x}, x_3, t) \in \mathbb{R}^{3+1}$, which is invariant under translations in direction of x_3. Compute the 2-dimensional Green's function using the 3-dimensional version, see (3.10).

ii) In 3 dimensions $u(\vec{x}_1, t_1)$ is completely determined by the values of $u(\vec{x}_2, t_2)$ through $(\vec{x}_1 - \vec{x}_2)^2 = c^2(t_1 - t_2)^2$ (light cone). What is changed in the 2-dimensional case?

2. Hertzian dipole

A Hertzian dipole is a time dependent point dipole $\vec{p}(t)$ with

$$\rho(\vec{x}, t) = -\vec{p}(t) \cdot \vec{\nabla}\delta(\vec{x}), \qquad \vec{\imath}(\vec{x}, t) = \dot{\vec{p}}(t)\delta(\vec{x}).$$

i) Verify the continuity equation.

ii) Compute the retarded electromagnetic potentials φ and $\vec{A}$ in Lorenz gauge. Compute the electromagnetic fields

$$\vec{E}(\vec{x}, t) = -\vec{\nabla}\varphi(\vec{x}, t) - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}(\vec{x}, t), \qquad \vec{B}(\vec{x}, t) = \operatorname{rot} \vec{A}(\vec{x}, t)$$

out of them. Order the contributions w.r.t. the powers of r^{-1}.

iii) Let the direction of $\vec{p}$ be constant. What are the directions of $\vec{E}$ and $\vec{B}$?

iv) Discuss which terms are dominating for $r \gg \lambda$ and $r \ll \lambda$ in the case of an electromagnetic wave which is harmonic in time (wavelength λ). What is the value of the relative phase between $\vec{E}$ and $\vec{B}$ in both limiting cases?

v) Compute the Poynting vector for $r \gg \lambda$, as well as the dependence of the power on the angle. What is the amount of the total power emitted?