ETH
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Statistical Physics
Exercise Sheet 9

HS 07
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Exercise 9.1 Bose-Einstein Condensation

a) The specific heat of a Bose gas viewed as a function of temperature has a cusp at $T = T_c$, the transition temperature. Thus, its derivative w.r.t. temperature is discontinuous. Calculate the magnitude Δ of this jump, i.e.,

$$\Delta = \lim_{T \to T_c^-} \partial_T C_V(T) - \lim_{T \to T_c^+} \partial_T C_V(T). \qquad (1)$$

Hint: Study the derivation of the formula for $C_V(T)$ in section 4.5.3. of the lecture notes first to become familiar with some tricks of the trade. As only the limit $T \to T_C$ is of interest you should make use of the expansion of $g_n(z)$ around $z = 1$. The expansion is

$$g_{5/2}(\nu) = 2.363\nu^{3/2} + 1.342 - 2.612\nu - 0.730\nu^2 + \dots, \qquad (2)$$

where $\nu = -\ln z$. Expansions for $g_{3/2}(\nu)$ etc. can be found using the recursion reation $g_{n-1}(\nu) = \partial_\nu g_n(\nu)$

b) At the transition, the compressibility $\kappa_T(T)$ of the gas diverges. Use the expansion of $g_n(z)$ around $z = 1$ keeping leading terms only to find the asymptotic behaviour of $\kappa_T(T)$ as $T \to T_c^+$ (You need only find the functional form without caring for constants).

Exercise 9.2 The Fate of a Hot Ball in Empty Space

Consider a ball in empty space that has an initial temperature T_0. Assuming that the ball is a black body, find its temperature as a function of time for the two cases that its specific heat is given by $C_V = \alpha T$ and $C_V = \alpha T^3$!

Exercise 9.3 A Simple Model of the Greenhouse Effect

a) Calculate the solar constant S_0 (energy flow density of the radiation of the sun on earth) using the following data: Radius of the sun $r_S = 6.96 \cdot 10^8 m$, Distance sun-earth $R = 1.50 \cdot 10^{11} m$, Stefan-Boltzmann constant $\sigma = 5.67 J s^{-1} m^{-2} K^{-4}$.

b) Using the result of a), calculate the earth's temperature looking for a stationary solution of the system. Model the earth as a black body and include the effect of reflection of the sun's radiation by modifying $S_0 \to (1 - r)S_0$. Consider the cases $r = 0$ and $r = 0.3$!

c) Building upon b), include the greenhouse effect by modeling the atmosphere as a layer around earth that is completely transparent for the sun's radiation, but absorbs all the radiation from earth (like the glass roof of a greenhouse).