Prof. H.R. Ott/M.Sigrist WS 05/06

Unkonventionelle Supraleitung

Serie 1

Verteilung: 1. November Abgabe: 8.November

1.1 Show that if the energy E, the volume V and the density $\rho=N/V$ of a system follow the relation $E = V f(\rho)$, the compressibility κ is given by

$$\kappa^{-1} = N\rho \frac{\partial \mu}{\partial N} \qquad \text{(Eq. II.8 of the experiment lecture note)}$$

1.2 From the Hamilton operator given in the Eq. 1.7 of the theory lecture note

$$H = \sum_{k,s} \xi_k c^+_{k,s} c_{k,s} + \sum_{k,k'} \tilde{V}_{kk'} c^+_{k,\uparrow} c^+_{-k,\downarrow} c_{-k',\downarrow} c_{k',\uparrow}$$

derive the gap equation, which for T near T_c can be written as

$$\Delta(\xi_k) = -N_0 \int_{\xi_{k'}>0} d\xi_{k'} \cdot \tilde{V}(\xi, \xi_{k'}) \frac{\tanh(\beta\xi_{k'}/2)}{\xi_{k'}} \Delta(\xi_{k'}) \qquad \text{(Eq. 1.10)}$$

where

$$\Delta_k \equiv -\sum_{k'} \tilde{V}_{k,k'} \langle c_{-k',\downarrow} c_{k',\uparrow} \rangle = \Delta(\xi_k) g_k \qquad \text{(Eqs. 1.8 and 1.9)}$$

and

$$\tilde{V}(\xi_k, \xi_{k'}) \equiv \int g^*_k \tilde{V}_{k,k'} g_{k'} \frac{d\Omega_k}{4\pi} \frac{d\,\Omega_{k'}}{4\pi} \qquad \text{(Eq. 1.11)}$$

Hints:

1) Approximate the potential-energy term in the Hamiltonian by replacing pairs of creation and annihilation operators by their average plus a term that may be considered to be “small”.
2) To diagonalize the resulting Hamiltonian use the Bogoliubov-Valatin transformation (see “Superconductivity” by R. D. Parks, Vol. I Chap. 2, Sect. 5, and/or “Introduction to Superconductivity” by M. Tinkham, Sect. 3.5 and 3.4, 3.6)

$$c_{k\uparrow} = u_k \gamma_{k0} + v^*_k \gamma^+_{k1}$$

$$c^+_{-k\downarrow} = -v_k \gamma_{k0} + u^*_k \gamma^+_{k1}$$

3) Show that

$$\langle c_{-k\downarrow} c_{k\uparrow} \rangle = u_k v^*_k [1 - 2f(E)] = \frac{\Delta_k}{2E_k} \tanh\left(\frac{\beta E_k}{2}\right)$$

4) And finally

$$\Delta_k = -\sum_{k'} V_{k,k'} \frac{\Delta_{k'}}{2E_{k'}} \tanh\left(\frac{\beta E_{k'}}{2}\right)$$