Übungsblatt III

Rückgabe: 18.3.2008

Aufgabe 1 [*Bound states in an (atomic) double-δ-potential*]: For the 1-dimensional potential given by $V(x) = -V_0\delta(x-a) - V_0\delta(x+a)$, (with $V_0 > 0$), determine the bound states, i.e. the solutions of the time-independent Schrödinger equation $\psi'' = \frac{2m}{\hbar^2}(V-E)\psi$ with $E < 0$.

- Split the real axis into three regions, $x < -a$, $-a < x < a$ and $a < x$, and solve the Schrödinger equation in each region separately.
- The above potential has the property that if ψ is a solution to $H\psi = E\psi$, it will have a definite *parity*, i.e. $\psi(-x) = \pm\psi(x)$. Split your solutions into sets of even (ψ_+) and odd (ψ_-) parity, and treat them separately.
- Now patch the solutions of the different regions together. Note that one can no longer demand continuity for ψ' (why?). To still extract jump conditions at the points where $\psi'_\pm$ is discontinuous, it is enlighting to integrate the Schrödinger equation over a small interval, say $[\pm a - \varepsilon, \pm a + \varepsilon]$ and take the limit $\varepsilon \to 0$.
- Derive an implicit equation for the quantity $\kappa = \sqrt{2m/\hbar^2|E|}$, and solve it graphically or numerically. Discuss the possibility of even and odd parity solutions $\psi_\pm$ depending on V_0 and a (the size of the molecule).
- Write down the possible energy eigenvalues and study the splitting of the energies in the limit $a \gg 1$.

Aufgabe 2 [*Tunnel effect, rectangular potential barrier*]: Study the scattering of a particle (i.e. the wavefunction ψ at energy $E > 0$) from the potential V with ($V_0 > 0$)

$$V(x) = \begin{cases} V_0 & -a < x < a\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

- Under the assumption that $0 < E < V_0$, make an ansatz for the solution of the time-independent Schrödinger equation, splitting the domain as before in regions I,II and III. Point out the difference to a classical treatment of the problem.
- From the continuity of ψ, ψ' at $x = \pm a$ derive the patching conditions.
- Now specialise to the case of an incident particle wave from the left (region I) and an outgoing wave to the right (III), i.e. consider the situation where

  $$\begin{aligned} \psi(x) &= e^{ikx} + re^{-ikx} \quad \text{for } x < a \\ \psi(x) &= te^{ikx} \quad \text{for } x > a. \end{aligned}$$

  Determine the transmission probability $T = |t|^2$ and reflection probability $R = |r|^2$ in the limit $a \gg 1$.