mechanical models for second law violation

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contents

outline

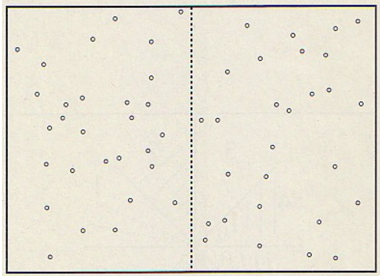
- the goal is to deduce the properties of mechanical models violating the second law

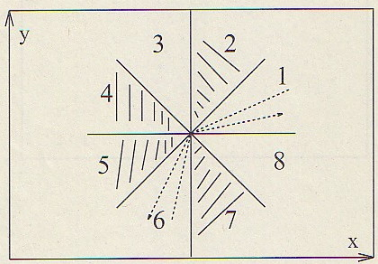
- finally we will see that:
  violation of the second law requires noninvariance of phase-space volume

model 1:

P.A. Skordos: Compressible dynamics, time reversibility, Maxwell's demon, and the second law

- operation is time-reversible
- speed and kinetic energy are conserved
- membrane more permeable from right than from left $\Rightarrow$ density difference
- equilibrium reached irreversibly when fluxes from both sides become equal

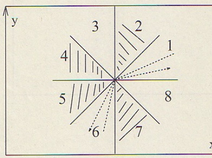


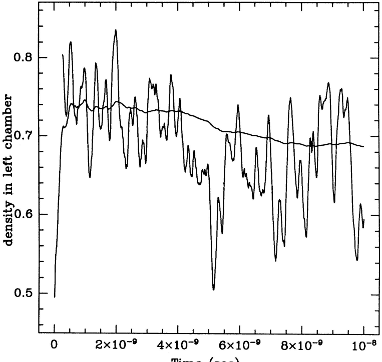


Estimate of density difference

- Assuming that the velocities of the disks are distributed isotropically $\Rightarrow$ (impact rate) $\propto |\cos\vartheta|$, ϑ=the impact angle
-

$$\Rightarrow \qquad N_L \approx 0.7 \qquad N_R \approx 0.3$$





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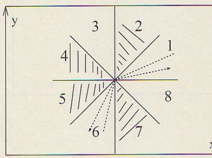

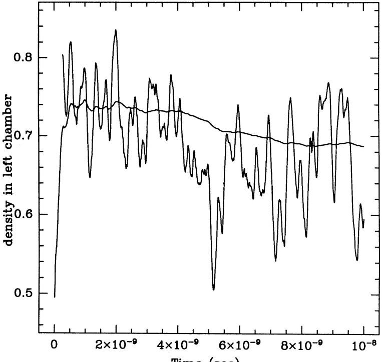

Compression of phase-space volume I

1. consider only one disk,characterized by (x, y, ϑ).Look at an infinitesimal phase volume $\omega(x_1, y_1, \vartheta_1)$ centred at a the point (x_1, y_1, ϑ_1).
   Assume: membrane at $x = 0$, $x_1 > 0$ near $x = 0$, $\vartheta_1 \in (\frac{3\pi}{4}, \pi)$.
   Also assume that after a time interval 1 that every point in $\omega(x_1, y_1, \vartheta_1)$ has moved to the left side of the membrane.
2. let $(\widetilde{x}, \widetilde{y}, \widetilde{\vartheta})$ denote the image of the evolution map

$$\widetilde{x} = -\sin\vartheta - x\frac{\sin\vartheta}{\cos\vartheta}$$

$$\widetilde{y} = y - x(1 + \frac{\sin\vartheta}{\cos\vartheta}) - \cos\vartheta$$

$$\widetilde{\vartheta} = \frac{3\pi}{2} - \vartheta$$

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other membrane maps

Is there another membrane map that would result in a density difference while preserving phase space volume?

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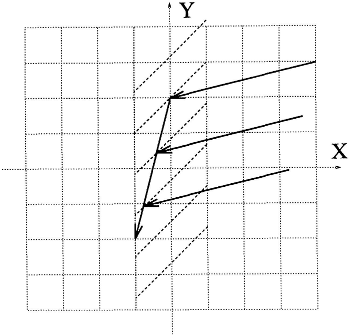

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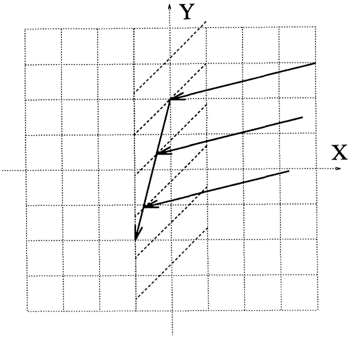


model 2:

Kechen and Kezhao Zhang: Mechanical models of Maxell's demon with noninvariant phase volume

mechanical models of Maxwell's demon I

- our starting point is a system of N interacting point particles:

$$m_i \ddot{\vec{r_i}} = \vec{F}(\vec{r_1}, \ldots, \vec{r_N}, \dot{\vec{r_1}}, \ldots, \dot{\vec{r_N}})$$

- we will use velocity-dependent force fields to implement a barrier which allows particles to pass through preferably only in one direction.We also demand:$\vec{F} \perp \vec{v}$ $\Rightarrow$the total energy is conserved.

$$\vec{F}(\vec{v}) = (\vec{v} \times \hat{\vec{z}}) A(v) B(\vartheta)$$

- $\vec{F}(\vec{v}) = (\vec{v} \times \hat{\vec{z}}) A(v) B(\vartheta)$ is time-reversible $\Leftrightarrow$ $\vec{F}(\vec{v}) = \vec{F}(\vec{-v})$
- phase volume conserved $\Leftrightarrow$ $B(\vartheta) = const$

mechanical models of Maxwell's demon II

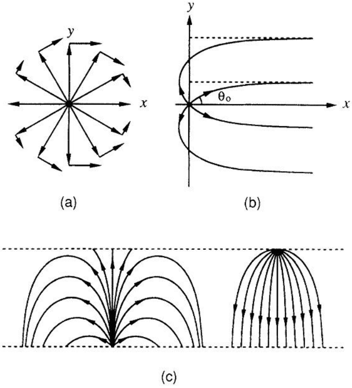


Figure: c)uniform field between dashed lines, downward: attracting direction, leakage caused by nearly perpendicular impinging

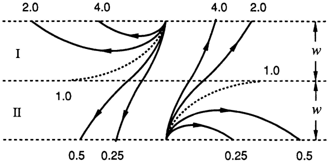


Figure: in I:attracting direction is upwards; in II: attracting direction is downwards; barrier width w chosen such that particles with speed v<1 pass downward, v>1 pass upward

is noninvariance of phase volume necessary for Maxwell's demon I

- let us introduce SMF(spontaneous momentum flow): sustaining and robust momentum flow inside an isolated mechanical system.
- there is a SMF in the system if the long-term average $\bar{J}_\nu = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau J_\nu(T_t \zeta) dt \neq 0 \quad \forall \zeta \in \Sigma(=\text{energy surface})$ for some spatial region V

  (where $J_\nu(\zeta) =$ total momentum inside V at a given time $= \sum_{i=1}^{N} p_i \chi_V(r_i)$)

is noninvariance of phase volume necessary for Maxwell's demon II

Formulation of the second law by SMF

existence of perpetual motion machine of the second kind $\Leftrightarrow$ existence of SMF

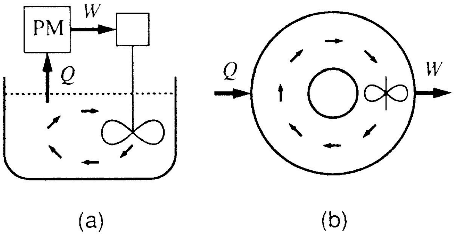


Figure: (a):$\Rightarrow$ (b)$\Leftarrow$

is noninvariance of phase volume necessary for Maxwell's demon III

Theorem: nonexistence of SMF

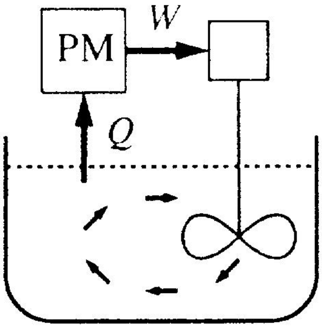
The point particle system $m_i\ddot{\vec{r_i}} = \vec{F}(\vec{r_1}, \ldots, \vec{r_N}, \dot{\vec{r_1}}, \ldots, \dot{\vec{r_N}})$ **cannot** exhibit SMF if

i) its energy function E is symmetric under momentum reversal, namely, $E(\zeta) = E(\tilde{\zeta})$ where $\zeta = (q_1, \ldots, q_s, p_1, \ldots, p_s)$ and $\tilde{\zeta} = (q_1, \ldots, q_s, -p_1, \ldots, -p_s)$

ii) phase volume $dq_1 \cdots dq_s dp_1 \cdots dp_s$ is invariant during time evolution, and total phase volume is finite for finite energy

is noninvariance of phase volume necessary for Maxwell's demon IV

Theorem⇒ such a system cannot serve as Maxwell's demon, because: suppose we can → contradiction to theorem



summary and conclusion

model 1

- time-reversible microscopic dynamics
- violation of second law $\Rightarrow$ compression of phase space

model 2

- time-reversible microscopic dynamics and purely mechanical
- In systems with symmetric energy wrt momentum reversal, 2^{nd} law violation requires noninvariance of phase volume.

ultimate result

The invariance of phase volume appears as a factor responsible for the validity of the second law of thermodynamics