Black Hole Thermodynamics, Hawking Radiation and Information Loss Paradox

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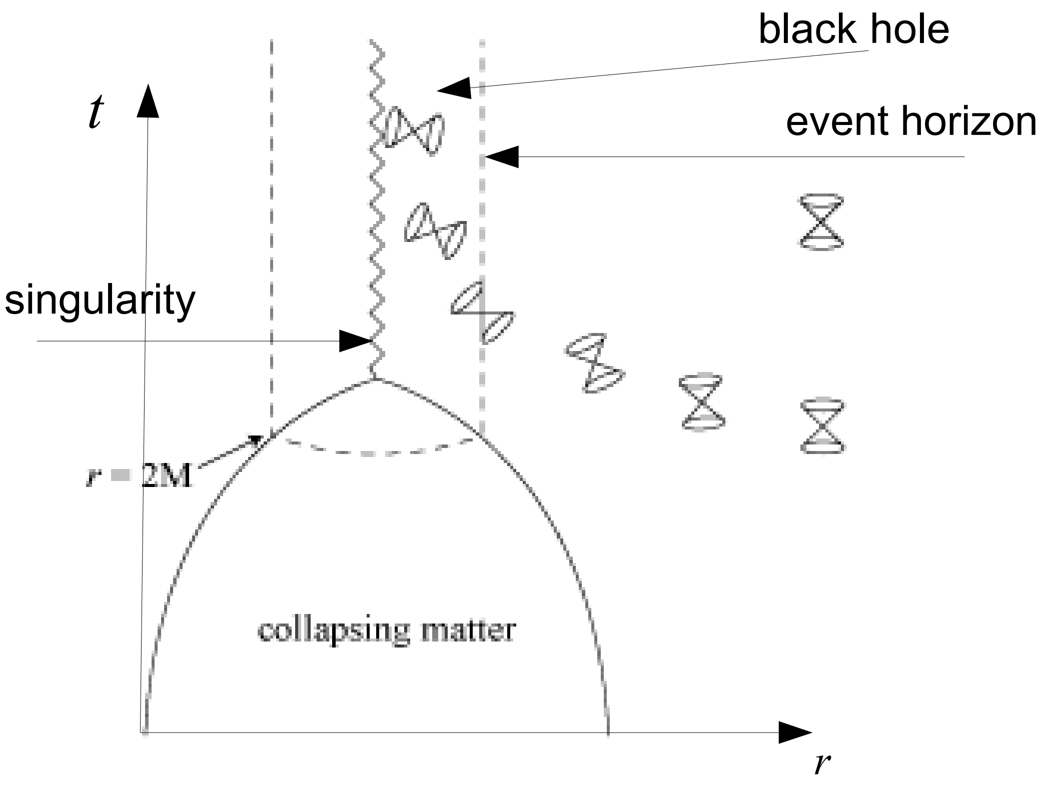
Contents

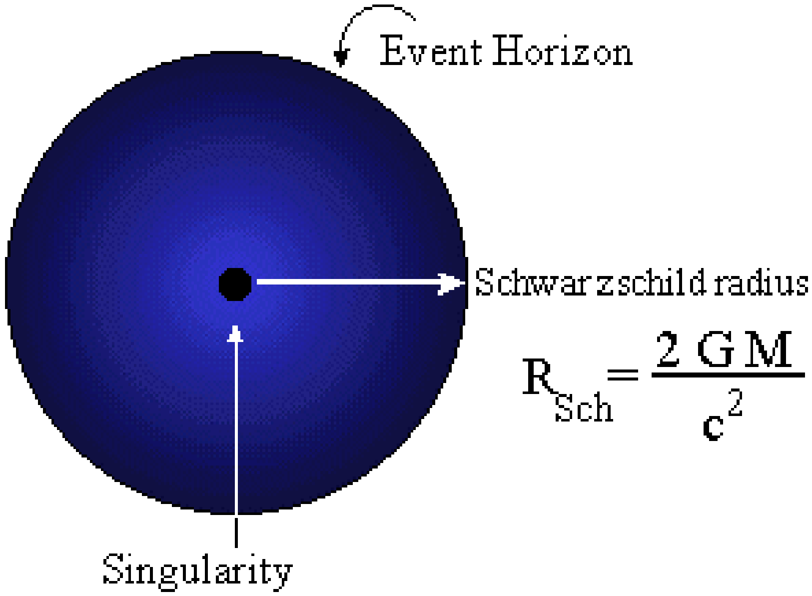
- Introduction
- The classical laws of black hole mechanics
- Hawking radiation theory/GSL
- Microscopic description of black hole entropy
- Information loss paradox and possible solutions
- Summary

Space-time diagram of spherical gravitational collapse - formation of a black hole:



<u>Black hole</u>: Region in space-time from which no signal can escape to infinity.

Structure of a spherical static black hole:



$$R_{Sch} = \frac{2GM}{c^2}$$

The surface gravity κ of a black hole is the acceleration needed to keep an object at the event horizon.

Stationary solutions of Einstein-Maxwell Equations

- Schwarzschild 1- parameter class of solutions:
  - Describes static spherically symmetric black holes with a mass M;

- Reissner-Nordstrøm 2-parameter class of solutions:
  - Describes black holes possessing electric charge Q and mass M;
  - Generalization of Schwarzschild's solutions; for Q=0 ⇒ Reissner-Nordstrøm → Schwarzschild;

- Kerr's 2-parameter class of solutions:
  - Describes black holes possessing mass M and angular momentum J; for J=0 ⇒ Kerr → Schwarzschild;

- Kerr-Newman's 3-parameter class of solutions:
  - Describes black holes possessing mass M, angular momentum J and electric charge Q;for Q=J=0 ⇒ Kerr-Newman → Schwarzschild;

No-Hair -Theorem:

"All black hole solutions of Einstein-Maxwell equations of gravity and electromagnetism in General Relativity can be comletely characterized by only three externally observable classic parameters: mass M, total angular momentum J and electric charge."

[Hawking, Israel, Robinson, Carter]

- <u>Consequences:</u> If there were other stationary solutions, they should form a 3-parameter family of solutions as well, depending only on the total mass M, the total angular momentum J and the electric charge Q.

Black hole uniqueness theorem:

"The Kerr-Newman family of solutions describes completely all the stationary black hole which can possibly occur in General Relativity.

[Robinson, 1973]

The classical laws of black hole mechanics

(Bardeen,Carter,Hawking,1973)

Close relationship between the laws of black hole mechanics and the laws of thermodynamics

The zeroth law of black hole mechanics:

“The surface gravity κ of stationary black hole is constant over the event horizon.”

The zeroth law of thermodynamics:

“The temperature T is constant for a system in thermal equilibrium.”

Are there other correspondences between black hole mechanics and thermodynamics?

Status in the early 70's:

- As a classical object with Temperature $T_{bh}=0$ it was assumed that $S_{bh}=0$ .If that were the case the second law of thermodynamics would be violated by entropy-laden matter entering the black hole, resulting in a decrease of the total entropy of the universe.

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Bekenstein,1972:

- Black holes must possess entropy $S_{bh}\sim A$ whose increase compensates the decrease of the exterior entropy such that the validity of OSL is preserved.

Motivation:

First law of black hole mechanics:

- Relates the the energy difference of two nearby black hole equilibrium states to the differences in the area A of the event horizons, in the angular momentum J, and in the charge

$$dM = \frac{\kappa}{8\pi} dA + \Omega\, dJ + \Phi\, dQ$$

First law of thermodynamics:

- $dU = TdS - pdV$

- The term $\Omega\, dJ + \Phi\, dQ$ represents the work done on the black hole by an external agent who increases the black hole's angular momentum and charge by dJ and dQ resp. Thus $\Omega\, dJ + \Phi\, dQ$ is the analog of $-pdV$,the work done on a TD system.

- Both laws are equivalent assuming: $S = \eta A$ and $T_{bh} = \epsilon \kappa$ such that $8\pi \eta \epsilon = 1$ for $\eta, \epsilon \in \mathbb{R}$.

- N.B.: The characteristic temperature T_{bh} differs from the effective temperature of the black hole which is zero.

Second law of black hole mechanics (Hawking's area theorem,1971):

The Area of the event horizon of each black hole does not decrease with time, i.e. $\delta A \geq 0.$

Second law of thermodynamics:

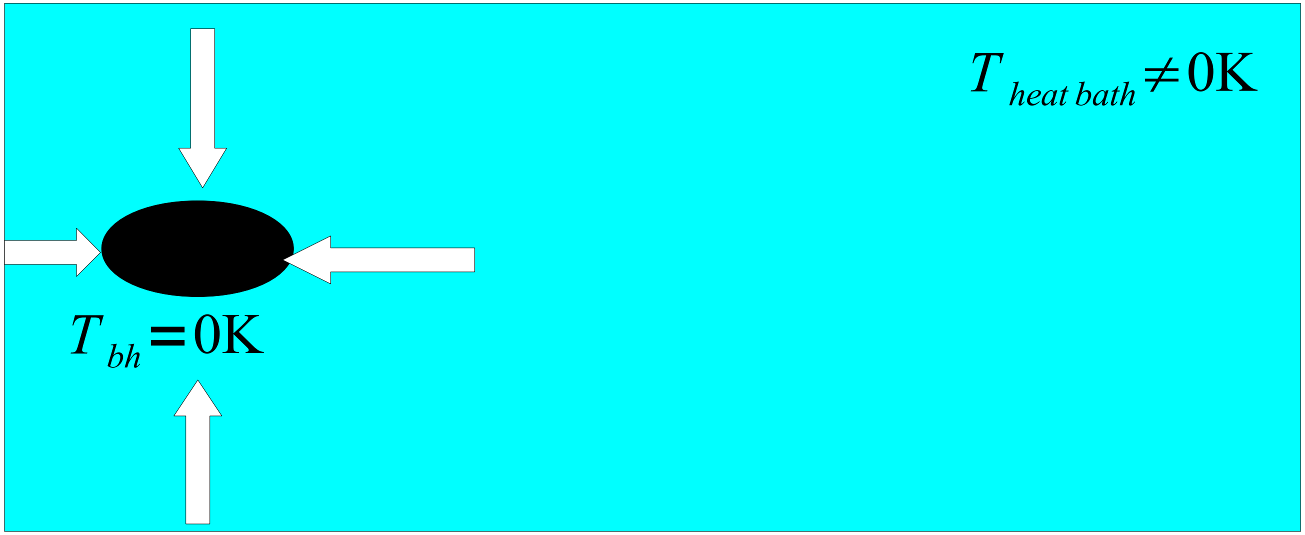
The Entropy of a closed system cannot decrease with time, i.e. $\delta S \geq 0.$

Remark:

- The second Law of black hole mechanics is slightly stronger than the corresponding thermodynamic law– one is allowed to transfer entropy from one system to another but one cannot transfer area from one black hole to another, thus the area of every individual black should not decrease.

Problem in the classical picture

<u>Immersing a classical black hole in a heat bath</u>:



The system will never reach a thermal equilibrium $\Longrightarrow$ breakdown of the classical thermodynamics in the presence of a black hole.

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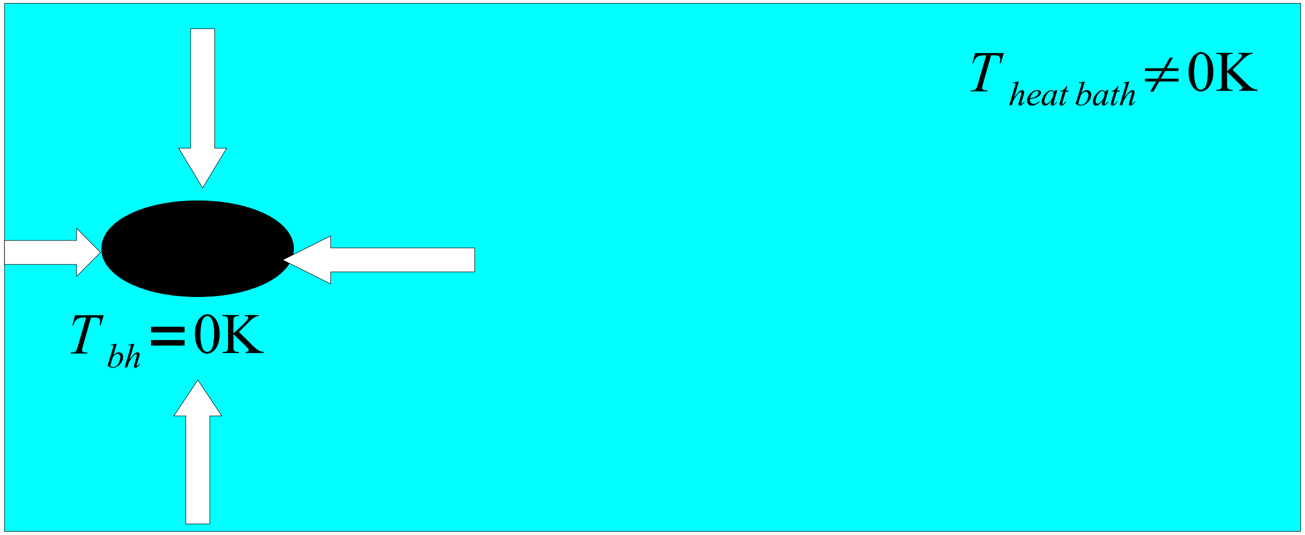

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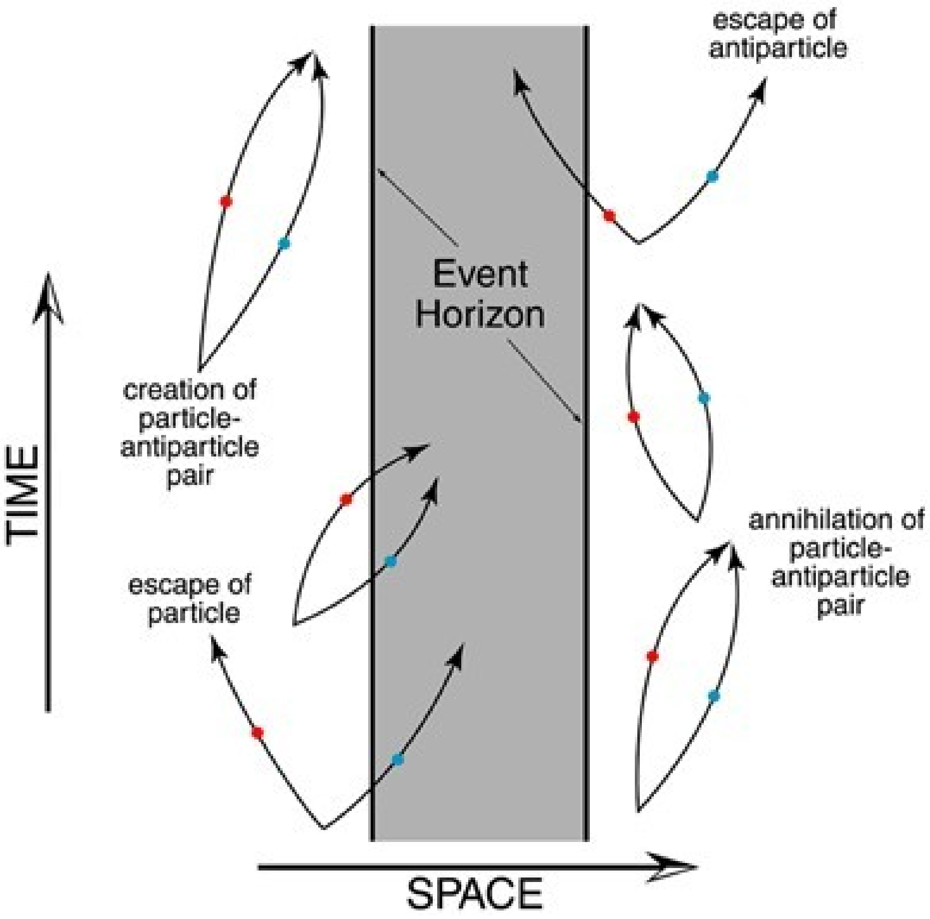
Conclusion: A black hole must have a non-zero temperature and it must emit thermal radiation!

Hawking radiation

(1974)

Hawking radiation :



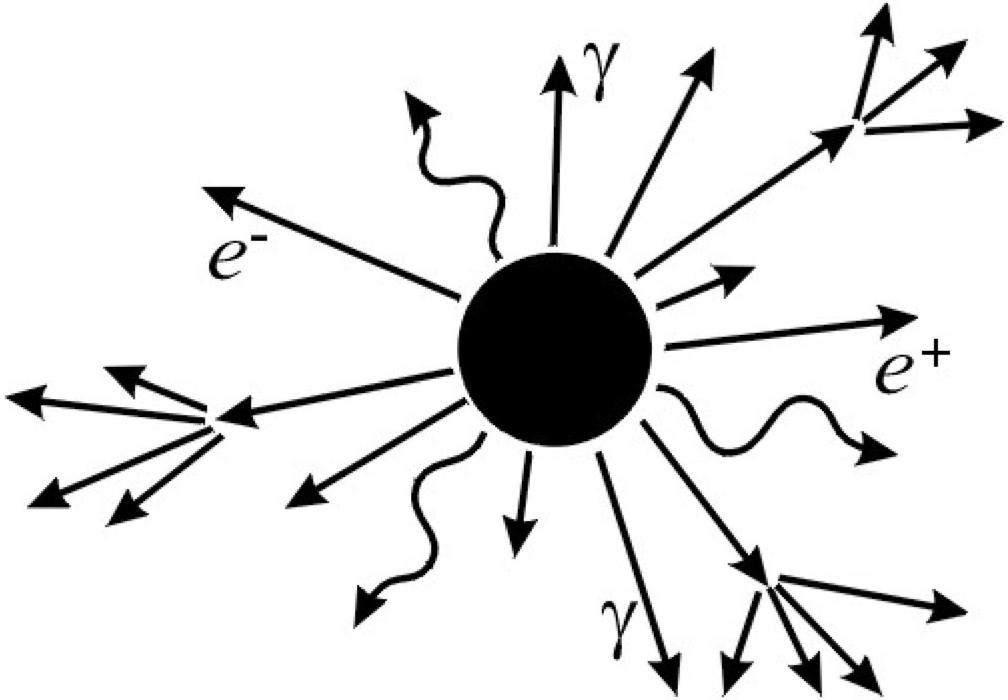
- Virtual particle-antiparticles are permanently created outside the event horizon.

- Three things can happen to such pairs:
  - Both particles are pulled into the black hole.
  - Both particles escape from the black hole.
  - One particle escapes while the other is pulled into the black hole.

- For the third possibility, the particle that has escaped becomes real and appears to have been emitted by the black hole. The anti-particle that was pulled into the black hole reduces the black hole mass, charge and angular momentum. As a result the black hole shrinks

Spectrum of radiation:

- The radiation spectrum contains all particles present in the matter theory .



- Hawking semiclassical calculation gives the expectation value $\langle N \rangle$ of the number of particles of a given species with charge q emitted in a mode with frequency ω and angular momentum m with reference to the axis of rotation:

$$\langle N \rangle = \Gamma \{ \exp[2\pi\kappa^{-1}(\omega - m\Omega - q\ \Phi)] \mp 1 \}^{-1}$$

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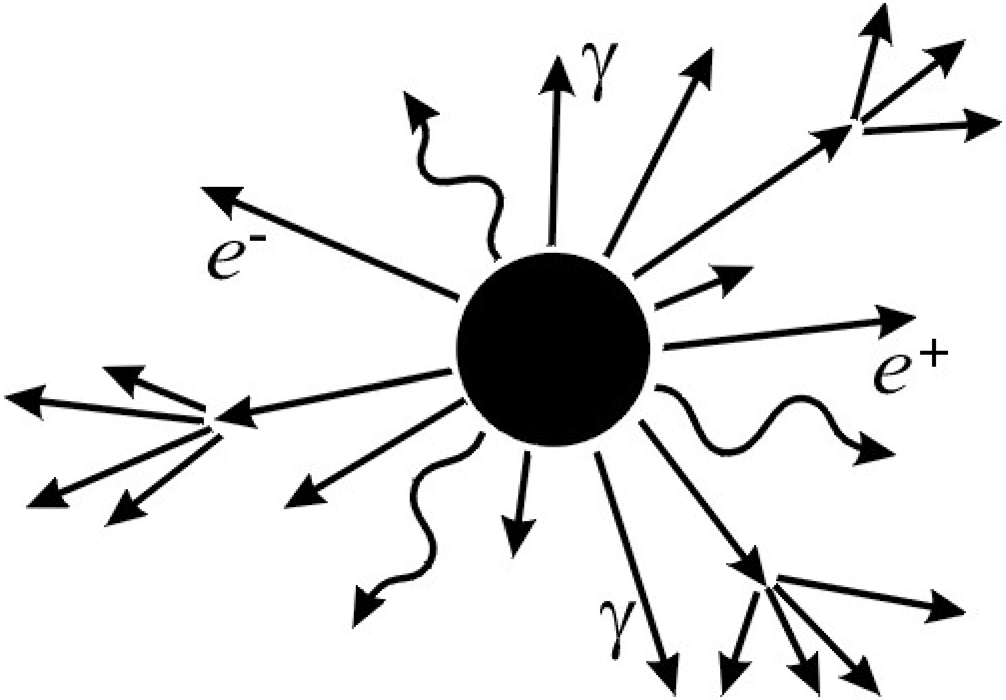

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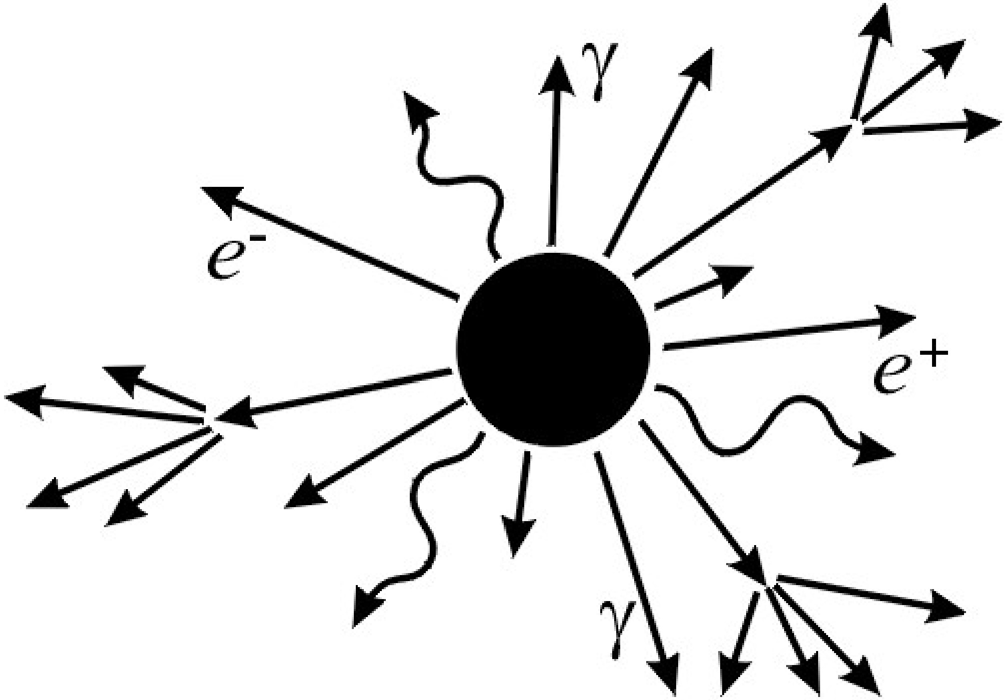

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Normalization factor

-1:bosons
+1:fermions

Remarks:

- Hawking's formula implies that more energetic modes have a lower probability of being emitted:

$$E \sim \omega$$

$$\langle N \rangle \sim \{\exp[2\pi\kappa^{-1}(\omega - m\Omega - q_i\Phi)] \mp 1\}^{-1} \underset{\omega\to\infty}{\rightarrow} 0$$

How can we understand this physically?

From Heisenberg's principle $\Delta E \, \Delta t \sim h$ follows that high energy particles have smaller lifetime , hence less time Δt for one of the particle-antiparticle pair to escape from the black hole.

Remarks:

- Hawking radiation implies that $T_{bh}=\frac{\kappa}{2\pi}$ is truly the physical temperature of a black hole, not merely a quantity playing a role mathematically analogous to temperature in the laws of black mechanics.

  Consequently, from $8\pi\eta\epsilon=1$ follows that $S_{bh}=\frac{A}{4}$ represents the physical entropy of the black hole.

- Hawking emission reduces the mass of the black hole and consequently the area A of its event horizon(since $A\sim M^2$) $\longrightarrow$ violation of the second law of black hole mechanics.

Generalized second law:

[Entropy of matter outside the black hole]+[Black hole entropy] never decreases.

[Bekenstein, 1972 / Hawking, 1974]

Microscopic description of black hole entropy

What does the black hole entropy represent microscopically?

- Jayne's information theory: The black hole entrpy is a measure for the number of internal configurations accessible to the system, i.e. $S=-\sum_n p_n \ln(p_n)$, where p_n is the probability for the n-th state accessible to the system to be occupied.

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Proposals:

- Could exist inside the black hole, in the singularity
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How should the degrees of freedom look like?

- One needs a quantum theory of gravity

Information Loss Paradox

How can one extract information from black holes - Do black holes destroy information?

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Information loss Paradox: An initially pure quantum state by collapsing to a black hole and then evaporating completely evolves to a mixed state, i.e. information gets lost.

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Violation of the unitary time evolution in quantum mechanics.

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Violation of the unitary time evolution in quantum mechanics.

Alternatives:

- Give up unitary-Reformulation of the foundations of quantum mechanics
- Try to find mechanism such that unitary is preserved and thus information conserved.

Information comes out with the Hawking radiation ?

Mechanism:Information is stored in the correlations between the quanta emitted early and the quanta emitted later on.

Drawback: Since the exterior of the black hole is not influenced by its interior, one has to explain how the black hole manages to record the information about the quanta that it has already emitted, so that it is able to induce these correlations.

The information is retained by a stable black hole remnant?

Mechanism: The black hole evaporates and decreases. When it reaches the Planck scale,quantum fluctuations dominate and the semi-classical calculation is no longer valid. The black hole does not evaporate completely, but a remnant remains which carries all the information that has fallen into the black hole.

Drawback:Since the initial black hole can be arbitrarily massive, the remnant must be capable of carrying an arbitrarily large amount of information($\approx M^2 / M^2_{Planck}$) . This is hard to integrate in the Quantum Field Theory (QFT).

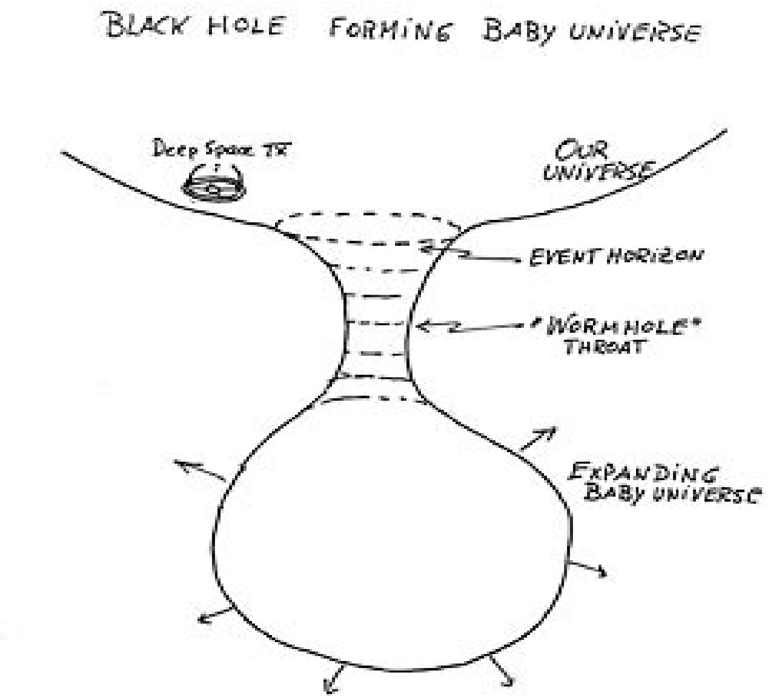
The information comes out “at the end”?

Mechanism:The radiation is thermal until the black hole reaches the Planck scale.The semi classical calculation breaks down. Instead of creating a stable remnant, information starts to leak out; it is in the correlations between the thermal quanta emitted earlier and the quanta emitted “at the end ” .

Drawbacks: The time for the Planck-size remnant to disappear is $t \approx M^4$ that is why there would be remnants that live arbitrarily long. We have the same problems as with absolutely stable remnants.

The information escapes to a baby universe?

<u>Mechanism</u>:



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<u>Drawbacks</u>:We do not have access to the information in our region of the “multiverse”.

Summary:

- Macroscopically black holes are completely defined by their mass, total angular momentum and total electric charge.
- Since black holes are” not completely black” there is a physical relation between surface gravity and temperature, black hole area and entropy .The zeroth and the first law of thermodynamics were found to hold for black holes,the second law was replaced by the GSL.
- We do not possess satisfactory microscopical description of the black hole entropy.
- Hawking's semi classical calculation led to the ILP which showed the incompatibility of principles of GR with the foundations of QM.
- We have to give up the idea of unitary in quantum mechanics or if we want to keep the classical QM we are challenged to find a mechanism by which information is not lost after it falls into a black hole.
- There are many solutions, but all of them have serious drawbacks. In order to analyze better these solutions one needs a quantum gravity theory, which we do not have at the moment.

Thank you for your attention!

Questions, please!

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