Exercises for "Phenomenology of Particle Physics II"

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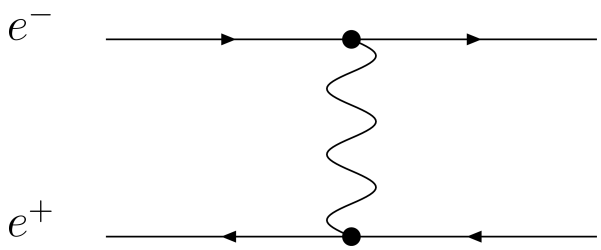
In this exercise we are concerned with QCD colour factors. We fix the notation as follows: The hermitian $N_C \times N_C$-matrices $T^a, a = 1, \ldots, N_C^2 - 1$ are a basis of the Lie Algebra of $SU(N_C)$. They have the properties

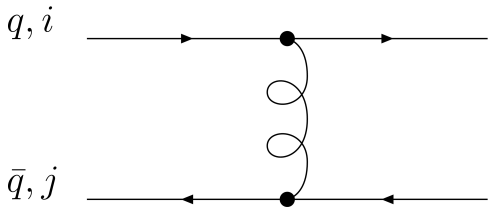
$$\begin{aligned} Tr(T^a) &= 0 \\ Tr(T^a T^b) &= \frac{1}{2}\delta_{ab} \quad \text{(Sometimes this normalization differs in the literature)} \\ \sum_a T^a_{ij} T^a_{kl} &= \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_C}\delta_{ij}\delta_{kl}\right) \end{aligned}$$

Exercise 7 Colour Factors for the QCD Potential

In QED, the interaction between an electron and a positron can be calculated to leading order in α by calculating the expectation value of the operator corresponding to

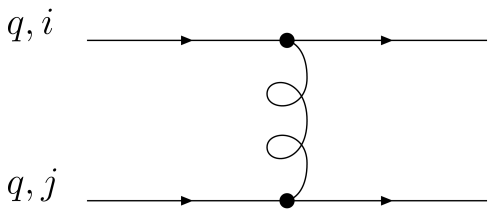


in the bound e^+e^- state. The diagram above corresponds to the well known $-\frac{\alpha}{r}$ potential of QED. In a QCD bound state the interaction is mediated by gluon exchange. The exchange of one gluon results in a $-f\frac{\alpha_s}{r}$ potential where f is a colour factor depending on the nature of the bound state.
Calculate the colour factor for the exchange of one gluon between a quark and an antiquark



$$\propto \sum_a T^a_{ki} T^a_{jl} \left|q_i\right\rangle \otimes \left|\bar{q}_j\right\rangle \left\langle q_k\right| \otimes \left\langle \bar{q}_l\right|$$

for a singlet quark-antiquark bound state $|q\overline{q}, \text{singlet}\rangle = \frac{1}{N_C} \sum_{m,n} |q_m\rangle \otimes |\overline{q}_n\rangle \, \delta_{mn}$ and for an octet quark-antiquark bound state $|q\overline{q}, b\rangle = \sqrt{2} \sum_{m,n} |q_m\rangle \otimes |\overline{q}_n\rangle \, T^b_{nm}$.
Calculate the colour factor for the exchange of a gluon between two quarks

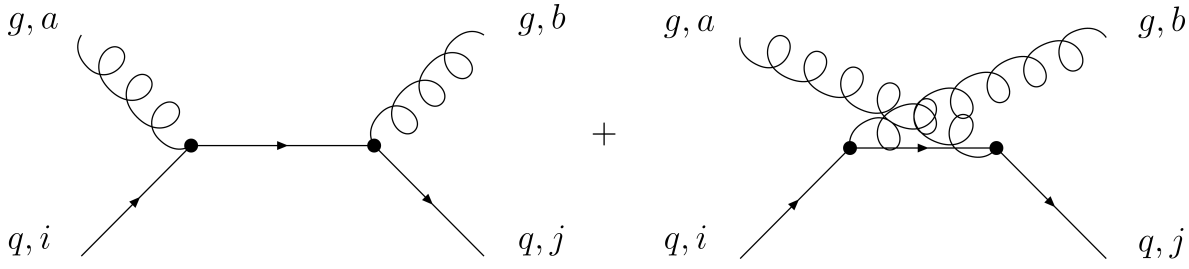


$$\propto \sum_a T^a_{i'i} T^a_{j'j} \left|q_i\right\rangle \otimes \left|q_j\right\rangle \otimes \left|q_k\right\rangle \left\langle q_{i'}\right| \otimes \left\langle q_{j'}\right| \otimes \left\langle q_k\right|$$

in a fully antisymmetric state of three quarks $|qqq, \text{antisymmetric}\rangle = \frac{1}{\sqrt{6}} \sum_{ijk} \epsilon_{ijk} \, |q_i\rangle \otimes |q_j\rangle \otimes |q_k\rangle$.

Exercise 8 Colour Algebra for Quark-Gluon Scattering

In the last semester we have calculated the matrix element for Compton scattering at tree level. The diagrams for this process are the same as for the scattering process $q + g \to q + g$ in QCD with the gluon taking the part of the photon.



The difference is that now the quark is in the fundamental representation of $SU(N_C)$ and the gluon in the adjoint representation. Calculate the colour factors that enter $|M|^2$ for this process, averaging over incoming colour indices and summing over outgoing colour indices.