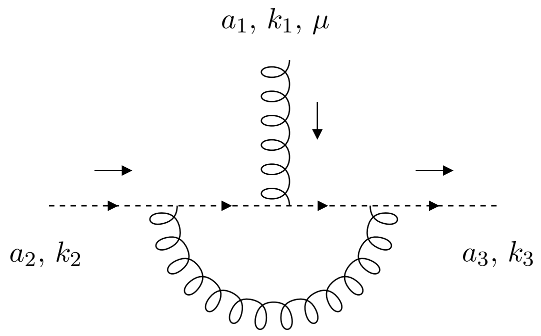
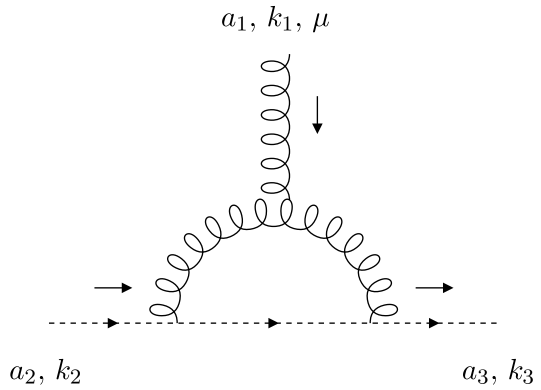
Quantum Field Theory II, Exercise Set 10.

FS 08/09

Ghost-Ghost-Gluon Interaction at One Loop

The goal of this exercise is at determine $\mathcal{Z}_1^{||}$ at one-loop order. We do this by verifying equations (8.104)–(8.107) in the lecture notes. The relevant Feynman diagrams are shown below. In the following we do not fix the gauge explicitly, i.e., the parameter a is arbitrary. As in the lecture we use dimensional regularization.





a) Write down the amplitudes of the two diagrams above.

We choose the standard basis of the fundamental representation of su(3) with $[t^a,\, t^b] = \mathrm{i} f^{abc} t^c$. Here $t^a_{ij} = \frac{1}{2}\lambda^a_{ij}$ are given by the Gell-Mann matrices λ^a. The corresponding adjoint representation is defined by $(F^a)_{bc} := \mathrm{i} f^{bac}$ and the commutation relations then read

$$[F^a,\, F^b] = \mathrm{i} f^{abc} F^c\,. \tag{1}$$

As usual we first treat the color contribution of these two diagrams:

b) Show that the amplitudes of these two diagrams are proportinal to $\frac{C_A}{2} f^{a_1 a_2 a_3}$.

Hint: Observe that the ampliudes are proportinal to $\mathrm{tr}(F^{a_1} F^{a_2} F^{a_3})$ *and* $\mathrm{tr}(F^{a_3} F^{a_2} F^{a_1})$*, respectively. Then use the identity* $F^a F^b = \frac{1}{2}[F^a,\, F^b] + \frac{1}{2}\{F^a,\, F^b\}$ *and show that terms proportional to anticommutators vanish.*

Since we are only concerned with the UV-divergent contribution of the above diagrams, we assume that the loop momentum is much bigger than any incoming momenta. Thus we might suppress the external momenta in the amplitudes in the following.

c) Show that both amplitudes are proportional to

$$\int \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{l^\mu l^\lambda}{l^6}, \tag{2}$$

where λ is a suitable Lorentz index.

After Wick-rotating to Euclidean space, this integral is UV-finite for $d - 4 < 0$. However, it is IR-divergent. Therefore we introduce an IR-cutoff Λ, i.e., we neglect momenta smaller than Λ (in Euclidean space).

d) Compute the integral (2) and express your result in terms of $2\varepsilon_{UV} := 4 - d$ and Λ.
Hint: Extract the The area of the 4-sphere is $2\pi^2$.

e) Verify equations (8.105) and (8.106) in the lecture notes by extracting the $1/\varepsilon$ divergences in your calculations.

It then follows immediately that, to one-loop order,

$$\mathcal{Z}_1^{||} = 1 - \frac{1}{2} C_A a \frac{g^2}{(4\pi)^2} \frac{1}{\varepsilon}. \tag{3}$$