Quantum Mechanics II
Exercise Sheet 6

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Exercise 6.1 The one-dimensional Lippmann-Schwinger equation

The goal of this exercise is the discussion of the Lippmann-Schwinger equation for one-dimensional scattering problems. Consider thus a particle that moves on the real line $\mathbb{R}$ which is exposed to a potential $V(x)$ which satisfies $xV(x) \to 0$ as $x \to \pm\infty$. As in the lecture we assume that there exist so called *stationary scattering waves* $\psi_k(x)$ with wave vector k of the time independent Schrödinger equation. These are states with the property

$$\psi_k(x) \sim e^{ikx} + f(k, k')e^{ik|x|} \tag{1}$$

($k' := k\,\mathrm{sgn}(x)$) as $x \to \pm\infty$.

a) Derive the Lippmann-Schwinger equation in this one-dimensional setting and compute the appearing Green's function.

b) Consider the special case of an attractive δ-function potential

$$V = -\left(\frac{\gamma\hbar^2}{2m}\right)\delta(x) \qquad (\gamma > 0). \tag{2}$$

Solve the integral equation to obtain the transmission and reflection amplitudes, T and R respectively, where these are defined by

$$\psi_k(x) = \begin{cases} T(k)e^{ikx}, & \text{as } x \to \infty \\ e^{ikx} + R(k)e^{-ikx}, & \text{as } x \to -\infty. \end{cases} \tag{3}$$