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Advanced Field Theory
Assignment 12

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Exercise 12.1 Derrick's Theorem

Prove that in a theory of scalar fields, there are no soliton solutions which are localized in more than one direction. We consider a theory with real scalar fields

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_i)(\partial^\mu \phi_i) - V(\phi_i), \qquad V(\phi_i) \geq 0.$$

We assume that there is a soliton solution $\phi_i(\mathbf{x})$ localized in D space dimensions with energy $E = T + U$,

$$T = \frac{1}{2}\int \mathrm{d}^D\mathbf{x}(\nabla\phi_i)^2, \qquad U = \int \mathrm{d}^D\mathbf{x}V(\phi_i).$$

1. Consider the rescaled soliton solution $\phi_i' = \phi_i(\mathbf{x}/\alpha)$. How does the energy depend on this parameter?

2. Argue why we need
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}E\bigg|_{\alpha=1} = 0.$$

3. Show that the above requirement cannot be fulfilled in $D \geq 2$.