Quantenmechanik II
Serie 11

FS 2010
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Aufgabe 11.1 Fockspace calculation

Verify that the scalar product in the Fockspace is given by (Eq. (10.44))

$$\langle \{n_{\mathbf{k},\lambda}\} | \{n'_{\mathbf{k},\lambda}\} \rangle = \prod_{\mathbf{k}\in\Lambda^*,\lambda=1,2} \delta_{n_{\mathbf{k},\lambda}, n'_{\mathbf{k},\lambda}}$$

using $a_{\mathbf{k},\lambda}|0\rangle = 0$, $\langle 0|0\rangle = 1$ and the commutation relation $[a_{\mathbf{k},\lambda}, a^\dagger_{\mathbf{k}',\lambda'}] = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\lambda,\lambda'}$.

Aufgabe 11.2 Second quantized form of free field operators

Use the Fourier series representation for $\mathbf{A}(\mathbf{x},t)$ (Eq. 10.21 with $c = 1$),

$$\mathbf{A}(\mathbf{x},t) = \sqrt{\frac{\hbar}{V}} \sum_{\mathbf{k},\lambda} \frac{\mathbf{e}_\lambda(\mathbf{k})}{\sqrt{2\omega(\mathbf{k})}} [a_{\mathbf{k},\lambda} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega(\mathbf{k})t)} + a^\dagger_{\mathbf{k},\lambda} e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega(\mathbf{k})t)}] \tag{1}$$

to verify that

a)

$$U = \frac{1}{2}\int (\mathbf{E}^2 + \mathbf{B}^2)\, d^3x = \frac{1}{2}\int (\dot{\mathbf{A}}\cdot\dot{\mathbf{A}} - \mathbf{A}\cdot\ddot{\mathbf{A}})\, d^3x = \sum_{\mathbf{k},\lambda} \hbar\,\omega(\mathbf{k})\,(a^\dagger_{\mathbf{k},\lambda} a_{\mathbf{k},\lambda} + \frac{1}{2})$$

b)

$$\mathbf{P} = \int (\mathbf{E}\wedge\mathbf{B})\, d^3x = -\int (\dot{\mathbf{A}}\wedge(\nabla\wedge\mathbf{A}))\, d^3x = \sum_{\mathbf{k},\lambda} \hbar\,\mathbf{k}\,(a^\dagger_{\mathbf{k},\lambda} a_{\mathbf{k},\lambda} + \frac{1}{2})$$

Aufgabe 11.3 Limit $L \to \infty$

Eq. (1) is defined in a box with $V = L^3$ and $\mathbf{k} \in \Lambda^*$ where $\Lambda^* = \{\mathbf{k}|\mathbf{k} = \frac{2\pi}{L}\mathbf{n}, \mathbf{n} \in \mathbb{Z}^3\}$ and the commutation relation is given by $[a_{\mathbf{k},\lambda}, a^\dagger_{\mathbf{k}',\lambda'}] = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\lambda,\lambda'}$. What form takes $\mathbf{A}(\mathbf{x},t)$ and the commutation relation when we let $L \to \infty$?

Aufgabe 11.4 Commutation relation of the electro-magnetic field

Show that $D_{ij}(t,\mathbf{x},\mathbf{y}) = [E_i(\mathbf{x},t), B_j(\mathbf{y},0)] = 0$ except when $(ct)^2 - (\mathbf{x}-\mathbf{y})^2 = 0$.