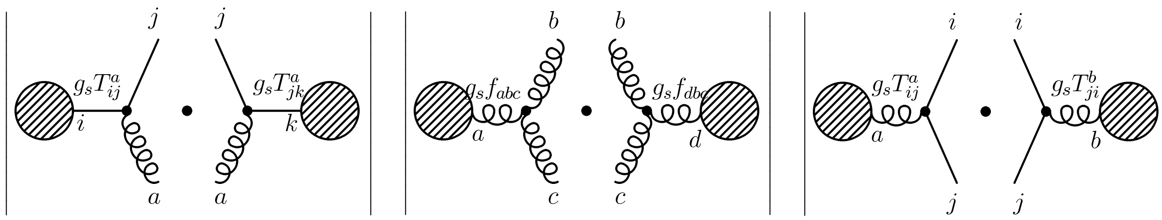
Sheet 5

Due: 12/04/11

Question 1 [*QCD colour factors*]: In this question we consider the square of a QCD amplitude with two external coloured legs. More specifically, consider the three squared amplitudes, where the shadowed blobs denote the remaining part of the Feynman diagrams.



By means of the QCD Feynman rules compute the corresponding colour factors and show that they are proportional to the $SU(N)$ Casimir operators $C_F = \frac{N^2-1}{2N}$, $C_A = N$, and the normalisation constant $T_R = \frac{1}{2}$, respectively.

Hint: You may use the Fierz identity for the representation matrices in the fundamental representation

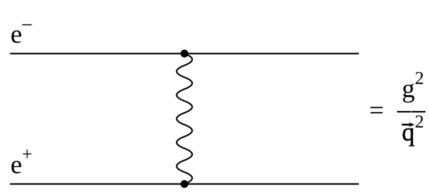
$$\sum_a T^a_{ij} T^a_{kl} = T_R \big(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}\big) \ . \tag{1}$$

In the adjoint representation you can use the relation

$$f^{abc} = -2i \mathrm{Tr}([T^a, T^b]T^c) \ , \tag{2}$$

where the trace Tr is taken in the fundamental representation.

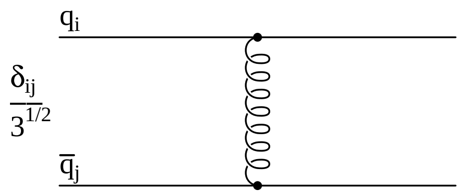
Question 2 [*Interquark QCD potential*]: In QED, at lowest order, the interaction between an electron and a positron in a bound state is represented by the following diagram



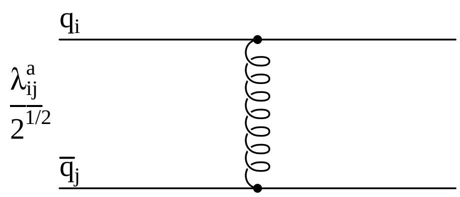
A photon is exchanged between electron and positron corresponding to the well known $-\alpha/r$ potential.

In QCD, the interaction between a quark and an antiquark is mediated by the exchange of a gluon. The potential is given by $V_{q\bar{q}} = -f\frac{\alpha_s}{r}$, where f is a colour factor which depends on the colour state of the quark-antiquark pair.

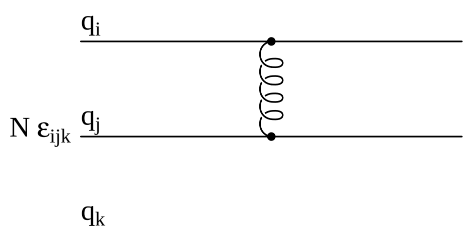
- Calculate the colour factor f for a color singlet quark-antiquark bound state



- Calculate the colour factor f for a colour octet quark-antiquark bound state, where $\lambda^a_{ij} = 2T^a_{ij}$,



- Calculate the colour factor f for the totally antisymetric state made of three quarks



where ϵ_{ijk} is the totally antisymmetric tensor.

Question 3 [*BRST Jacobian*]: For a two-dimensional integral of the form

$$\int dy d\eta f(y, \eta) \tag{3}$$

where y is a bosonic degree of freedom while η is a Grassmann variable, consider the BRST-like transformation

$$\begin{aligned} y &= x + \lambda a(x, \xi) \\ \eta &= \xi + \lambda b(x, \xi) \ , \end{aligned} \tag{4}$$

where λ is a Grassmann variable, *i.e.* $\lambda^2 = 0$. Determine the Jacobian of the transformation.

Hint: Perform the transformation (4) in the integral (3)

$$\int dy d\eta f(y, \eta) = \int dx d\xi f(x + \lambda a(x, \xi), \xi + \lambda b(x, \xi)) J(x, \xi). \tag{5}$$

Now expand both sides of (5) in λ, and show that the Jacobian $J(x, \xi)$ must have the form $J(x, \xi) = 1 + \lambda j(x, \xi)$. Then determine $j(x, \xi)$ by comparing the two expansions. A generalisation of this argument to higher dimensional (path) integrals can be used to show that the path integral measure is actually BRST-invariant.