Sheet 8

Due: 17/05/11

Question 1 [*The Standard Model Higgs mechanism: Lepton masses*]:

(i) First of all, let us see why we need the Higgs mechanism at all. As an example we consider the first lepton generation, i.e. the electron and its neutrino. The gauge group is $SU(2) \times U(1)$. From experiments, we know that there are no *right-handed neutrinos* in nature (neglecting neutrino oscillations). Therefore, right- and left-handed leptons appear in the Lagrangian with different structures. The left-handed electron and neutrino form a doublet w.r.t the gauge group $SU(2)$:

$$\psi_L := \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \mapsto \begin{pmatrix} \nu'_L \\ e'_L \end{pmatrix} = e^{i\theta^a T^a} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \tag{1}$$

where the $T^a = \sigma^a/2$ are the generators of $SU(2)$ (σ^a are the Pauli matrices). The right-handed electron, on the other hand, forms a singlet under $SU(2)$, i.e. it is $SU(2)$ invariant:

$$\psi_R := e_R \mapsto e'_R = e_R. \tag{2}$$

Under $U(1)$, the two structures transform as

$$\psi_L \mapsto \psi'_L = e^{-i\theta}\psi_L, \quad \psi_R \mapsto \psi'_R = e^{-2i\theta}\psi_R. \tag{3}$$

If we now naively tried to give a mass to the electron, we would include the following term in the Lagrangian:

$$\mathcal{L} \supset -m\bar{e}e = -m(\bar{e}_L e_R + h.c.). \tag{4}$$

Why is this term not allowed?

(ii) To assign mass in a gauge-invariant way, we now introduce a complex *scalar field* H which couples to fermions as follows:

$$\mathcal{L} \supset y\bar{\psi}_L H \psi_R + h.c., \tag{5}$$

where y is a coupling constant. How does H have to transform under $SU(2)$ and $U(1)$?

(iii) Now that you've shown that H is a $SU(2)$ doublet, find the gauge transformation i.e. the $SU(2)$ matrix U such that

$$UH = U \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} 0 \\ H_r \end{pmatrix} \tag{6}$$

where H_r is real. This is called the *unitary gauge*.

(iv) We add (in addition to the usual kinetic terms) a potential for the scalar field:

$$\mathcal{L} \supset -\mu^2(H^\dagger H) - \frac{\lambda}{4}(H^\dagger H)^2. \tag{7}$$

If $\mu^2 > 0$, the potential has a minimum at $H_r = 0$ and the vacuum expectation value (VEV) of the Higgs field is $v := \langle H_r \rangle = 0$. Show that $v \neq 0$ for $\mu^2 < 0$.

(v) Choosing $\mu^2 < 0$, we can now write $H_r = v + h$ (with $\langle h \rangle = 0$), where h corresponds to the notorious physical Higgs boson. Expand the fermion mass term in Eq. (5) and show that we've assigned a mass to the electron in a gauge invariant way. What is the mass expressed in terms of v and y? In addition, we obtain an interaction between the Higgs boson and the electron. What is the interaction's coupling strength expressed in terms of m_e (the electron mass) and v?

Question 2 [*Higgs couplings in the standard model*]:

Starting from the Lagrangian density

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi), \qquad D_\mu = \partial_\mu - igT^aW_\mu^a - ig'\frac{Y}{2}B_\mu, \quad V(\phi) = \mu^2\phi^2 - \frac{\lambda}{4}\phi^4 \tag{8}$$

for the scalar doublet

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \tag{9}$$

find the couplings hWW,$hhWW$, hZZ and $hhZZ$.
Hints:

(i) Consider the Lagrangian density with $Y = 1$, $T^a = \frac{1}{2}\sigma^a$ and use the explicit form of the Pauli matrices ($\sigma^1 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right), \sigma^2 = \left(\begin{smallmatrix} 0 & -i \\ i & 0 \end{smallmatrix}\right), \sigma^3 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$).

(ii) Diagonalize the quadratic terms by introducing the physical fields

$$W_\mu^+ = \frac{1}{\sqrt{2}}\left(W_\mu^1 - iW_\mu^2\right) = (W_\mu^-)^\dagger \tag{10}$$

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \tag{11}$$

$$A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}}. \tag{12}$$

(iii) Now you can read off the coefficients in the expansion

$$(D_\mu\phi)^+(D_\mu\phi) = (\partial_\mu h)^2 + M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu - iV_{hWW}hW_\mu^+W^{-\mu} \tag{13}$$

$$-iV_{hhWW}hhW_\mu^+W^{-\mu} - iV_{hZZ}hZ_\mu Z^\mu - iV_{hhZZ}hhZ_\mu Z^\mu. \tag{14}$$