Exercises: Lecture 2

1) With two massless flavors, the QCD Lagrangian is invariant under an $U(2)_L \times U(2)_R$ chiral symmetry group. What if the QCD color group was $SO(3)_c$ instead of $SU(3)_c$? What the global flavor group would be in that case?

   Hint: By charge conjugation I can define $q_L^c \equiv (q_R)^c$, and I could consider, in QCD, more general unitary transformations that mix q_L with q_L^c. Think to why these transformations are not actually allowed in QCD.

2) The Decay process $\pi^+ \to \mu^+ \nu_\mu$ is mediated by the Fermi Lagrangian

$$\mathcal{L}_{FERMI} = \frac{G_{WK}}{\sqrt{2}} \left(V_+^\mu + A_+^\mu\right) \sum_{\ell = e,\mu} \bar{\ell}\, \gamma_\mu (1+\gamma_5) \nu_\ell + h.c.$$

$$V_\pm = V_1 \pm i V_2 \qquad A_\pm = A_1 \pm i A_2$$

$$G_{WK} = 1.15 \cdot 10^{-5}\, GeV^{-2}$$

Compute $\Gamma(\pi^+ \to \mu^+ \nu_\mu)$ and show that

$$\Gamma(\pi^+ \to \mu^+ \nu_\mu) = \frac{G_{WK}^2 F_\pi^2 m_\mu^2 (m_\pi^2 - m_\mu^2)^2}{16\pi m_\pi^3}$$

From the above formula, and using the measured value

$$\Gamma = (2.6 \times 10^{-8} s)^{-1}$$

extract the value of F_π.

Also, why is the decay to <u>electrons</u> suppressed?

3) Show that the most general form of the neutron-proton matrix element of the axial current, compatibly with Lorentz, P and C invariance, is:

$$\langle p | A_\mu^+(0) | n \rangle = \bar{u}_p \left[f[q^2] \gamma^\mu \gamma_5 + g(q^2) q^\mu \gamma_5 \right] u_n$$

where $q = P_n - P_p$ and $u_{n,p}$ are neutron's and proton's wave functions.

<u>Hint</u>: See Weinberg 10.6