Exercises: Lecture 3.

1) Show that the $SU(2)_L \times SU(2)_R$ group has the same algebra as $SO(4)$.

<u>Hint</u>: Given the explicit form of the (2, 2) representation in terms of the 4-plet h_n:

$$\phi = h_4 \mathbb{1} + i h_i \sigma^i$$

Compute explicitly the 6 generators as 4x4 matrices on h_n. Show that they span the $SO(4)$ algebra in the fundamental representation

2) Check that by the field redefinition

$$h_n = R_{n4}\,\sigma \;; \quad R_{a4} = \frac{2\,\varphi_a}{1+\vec{\varphi}^2} \;; \quad a = 1, 2, 3$$

the $SO(4)$ linear σ-model Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + 2\sigma^2 \vec{D}_\mu \cdot \vec{D}^\mu + \frac{\mu^2}{2}\sigma^2 - \frac{\lambda}{4}\sigma^4$$

$$\vec{D}_\mu \equiv \frac{\partial_\mu \vec{\varphi}}{1+\vec{\varphi}^2}$$

3) Given the action δh_M, studied in ex. 1, of the broken and unbroken generators on the 4-plet h_M, derive their action $\delta \xi^a$ on the ξ^a fields, as defined in exercise 2 and in the Lecture. Show that $\vec{D}_\mu$ undergoes a <u>linear</u> but <u>field-dependent</u> SO(3) isospin rotation under the broken generators.

<u>Hint</u>: Look for help in Weinberg pg 194-195