Exercises: Series 5

1) In the Scalar Theory with shift symmetry, defined and discussed during the lecture, compute:

1) The Feynmann Rules for the $g(4,4)$, $g(4,6)$ and $g(4,8)$ vertices. Both for generic off-shell[1,2] momenta and in the on-shell case

2) The 1-loop amplitude for $2 \to 2$ mediated by two $g(4,4)$ insertions. Of course, the one before the integral given during the lecture

3) Try to compute the other contributions to the Λ^2 divergence, the ones from the denominator not computed during the lecture. Do you manage to show that it renormalises $g(4,6)$?

2) Show, using

2) Show, using Lorentz invariance, the identities

$$\int d^4p \, p_\mu p_\nu f(p^2) = \frac{1}{4} \eta_{\mu\nu} \int p^2 f(p^2) \, d^4p$$

$$\int d^4p\, p_\mu p_\nu p_\rho p_\sigma f(p^2) = \frac{1}{24}\left[\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\right] \times \int d^4p\, p^4 f(p^2)$$