Advanced Topics in Quantum Information Theory
Exercise 4

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Exercise 4.1 Error Models in the Lindblad Picture

Under certain conditions the evolution $\rho(t)$ of an open quantum mechanical system can be described by the Lindblad equation

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_i \gamma_i \left(c_i \rho(t) c_i^\dagger - \frac{1}{2} c_i^\dagger c_i \rho(t) - \frac{1}{2} \rho(t) c_i^\dagger c_i \right) , \tag{1}$$

where H denotes the ordinary system Hamiltonian, the c_i are the Lindblad operators, and the γ_i non-negative constants. In the following we set $H \equiv 0$ and solve (1) for different scenarios.

a.) Consider one qubit and Lindblad operators $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\mathbb{1}_2$ with constants $\gamma_{\sigma_z} = \gamma_{\mathbb{1}_2}$ respectively. This regime corresponds to the error model 'dephasing', where only phase flip errors happen. Solve (1) for this case. What happens for $t \to \infty$? Generalize your results to K qubits, where every single qubit can undergo a phase flip error independently (total dephasing). Is there a decoherence free subspace? Is it possible to store classical information reliably in such a system?

b.) Consider one qubit and Lindblad operators $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\mathbb{1}_2$ with constants $\gamma_{\sigma_x} = \gamma_{\sigma_y} = \gamma_{\sigma_z} = \gamma_{\mathbb{1}_2}$ respectively. This regime corresponds to an error model, where bit flip errors, phase flip errors or both happen. Solve (1) for this case. What happens for $t \to \infty$? Generalize your results to K qubits, where every single qubit can undergo a bit flip error, a phase flip error or both independently. Is there a decoherence free subspace? In the lecture we called this error model total decoherence. Compare your results with the conclusion drawn in the lecture.

c.) Consider two qubits and and Lindblad operators $\sigma_x \otimes \mathbb{1}_2 + \mathbb{1}_2 \otimes \sigma_x$, $\sigma_y \otimes \mathbb{1}_2 + \mathbb{1}_2 \otimes \sigma_y$, $\sigma_z \otimes \mathbb{1}_2 + \mathbb{1}_2 \otimes \sigma_z$, and $\mathbb{1}_2 \otimes \mathbb{1}_2$ with constants $\gamma_{\sigma_x} = \gamma_{\sigma_y} = \gamma_{\sigma_z} = \gamma_{\mathbb{1}_2}$ respectively. This regime corresponds to an error model, where both qubits undergo the same bit flip error, phase flip error or bit and phase flip error combined. Solve (1) for this case. What happens for $t \to \infty$? Is there a decoherence free subspace? This is the simplest case of the collective decoherence. Compare your results with the conclusion drawn in the lecture.

Exercise 4.2 Collective Decoherence

Generalize the collective decoherence model of Exercise 4.1 c.) from two to K qubits and use the results from the lecture to compute the decoherence free subspace when K is even.