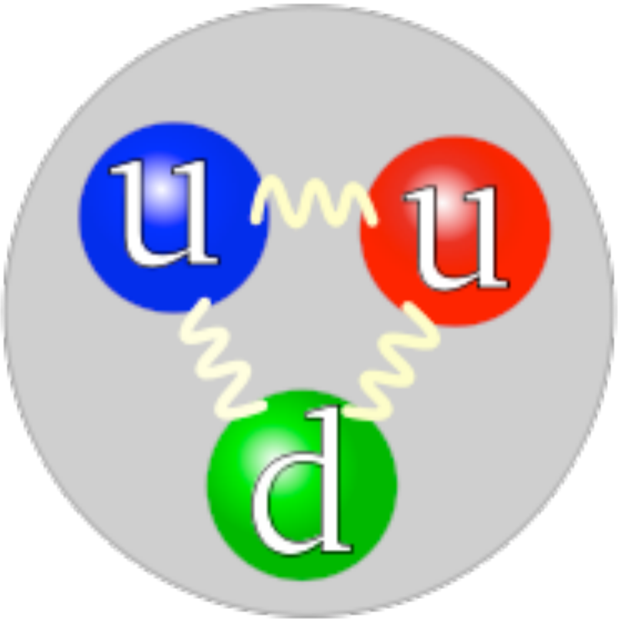

Structure of Hadrons and the parton model

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Phenomenology of Particle Physics - FS2011

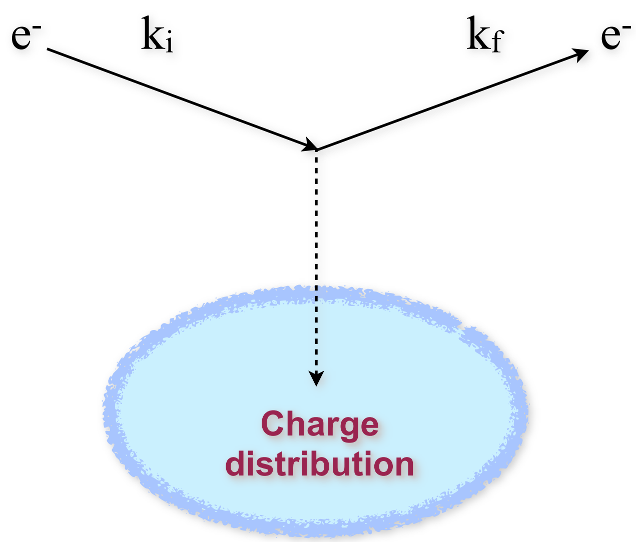
Lecture: 10/5/2011

Topics in this lecture

- How do we study the structure of composite particles?
- Is the proton an elementary particles?
- If not, what do we see inside the proton?
- Are there only charged partons inside the proton?

Probing a charge distribution

- To probe a charge distribution in a target we can scatter electrons on it and measure their angular distribution
- The measurement can be compared with the expectation for a point charge distribution



Form factor

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{point}} |F(q)|^2$$

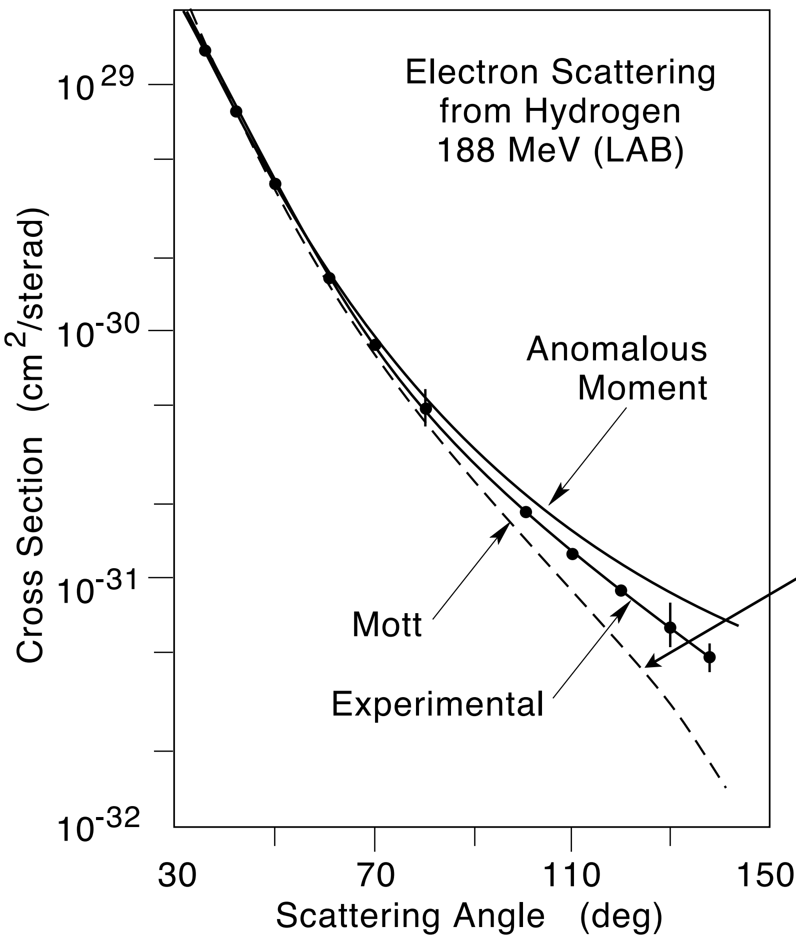
Momentum transfer:

$$q = k_i - k_f$$

Structureless target:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{point}} = \frac{(Z\alpha)^2 E^2}{4k^4 \sin^4(\theta/2)}\left(1 - \frac{k^2}{E^2}\sin^2\theta/2\right)$$

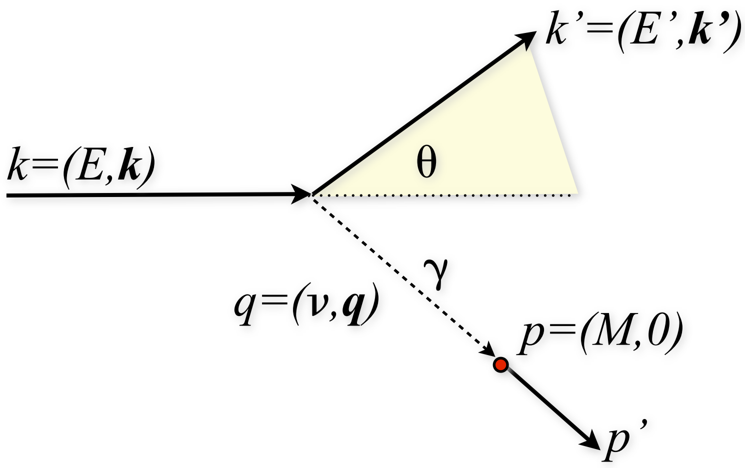
Proton is not a “point”



$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} = \frac{(Z\alpha)^2 E^2}{4k^4 \sin^4(\theta/2)} \left(1 - \frac{k^2}{E^2} \sin^2 \theta/2\right)$$

Structureless point-like target does not describe the data!

e^--μ scattering in the lab frame



1) Matrix element

$$|\bar{M}|^2 = \frac{e^4}{q^4} L_e^{\mu\nu} L_{\mu\nu}^{muon}$$

$$|\bar{M}|^2 = \frac{8e^4}{q^4} 2M^2 E' E \left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

2) Transferred momentum

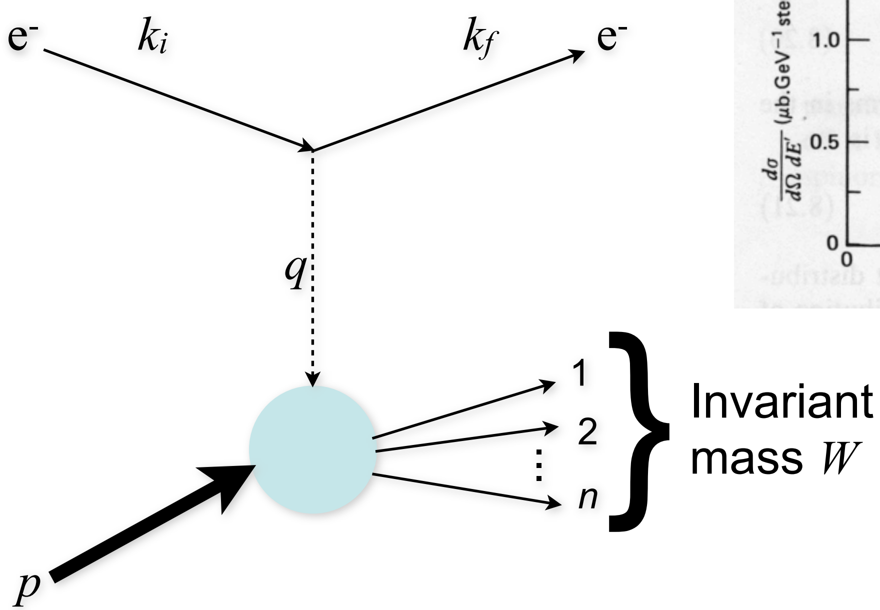
$$q^2 \simeq -2k\cdot k' \simeq -4EE'\sin^2\frac{\theta}{2}$$

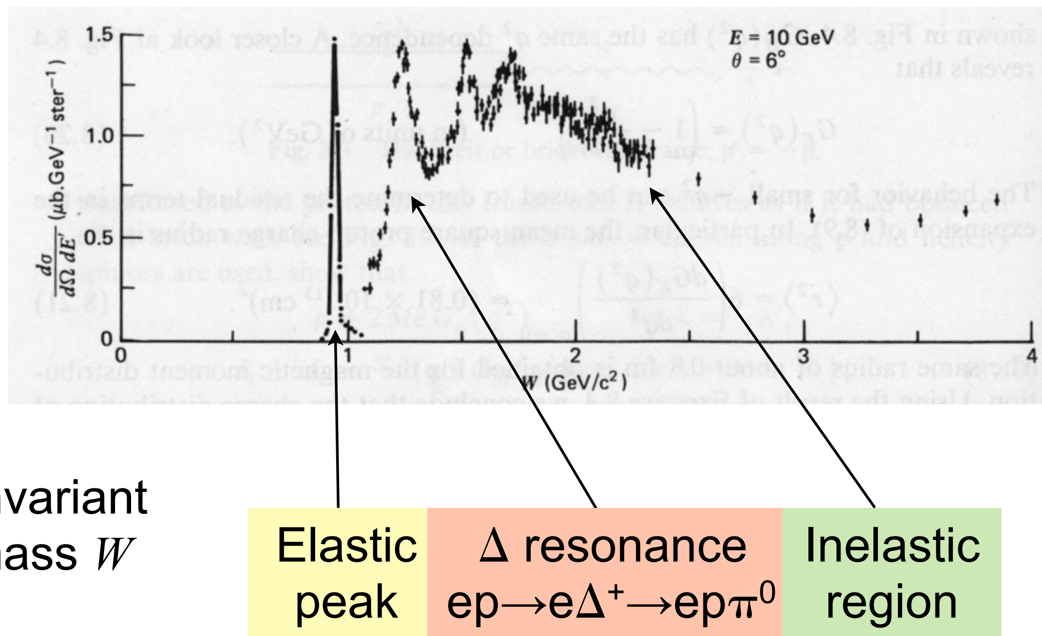
$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

Target recoil (↑ $\frac{E'}{E}$)

Electron-proton scattering

- The scattering picture used so far needs to be extended for a *composite* object
- The invariant mass spectrum shows the elastic peak, excited baryons followed by an inelastic smooth distribution





Hadronic tensor - 1

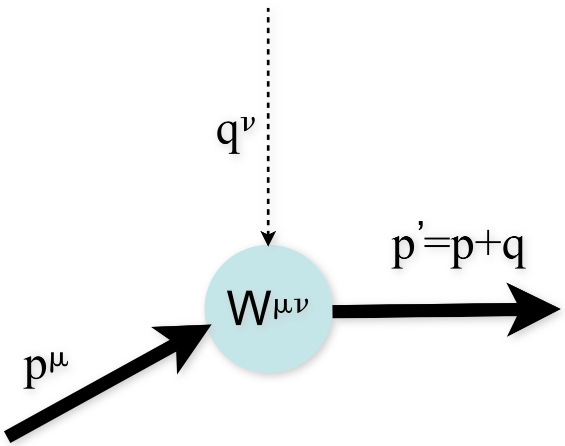
$$d\sigma \sim L^{e}_{\mu\nu} L^{\mu\nu}_{muon} \rightarrow d\sigma \sim L^{e}_{\mu\nu} W^{\mu\nu}_{proton}$$

electron-muon scattering

parametrizes the current at the proton vertex

- The most general form of the tensor W depends on $g^{\mu\nu}$ and on the momenta p and q

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2}(p^\mu q^\nu + q^\mu p^\nu)$$



- From current conservation $\partial_\mu J^\mu = 0$

$$W_5 = -\frac{p\cdot q}{q^2} W_2$$

$$W_4 = \left(\frac{p\cdot q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1$$

Hadronic tensor - 2

- Only two independent inelastic ***structure functions (W_1 and W_2)***

$$W^{\mu\nu} = -W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)$$

- Each structure function has two independent variables

q^2 — *square of transferred four-momentum*

$\nu \equiv \frac{p\cdot q}{M}$ — *energy transferred to the nucleon by the scattering electron*

Dimensionless variables

$x = \frac{-q^2}{2p\cdot q} = \frac{-q^2}{2M\nu}$ $\qquad 0 \le x \le 1$ ***Bjorken scaling variable***

$y = \frac{p\cdot q}{p\cdot k}$ $\qquad 0 \le y \le 1$

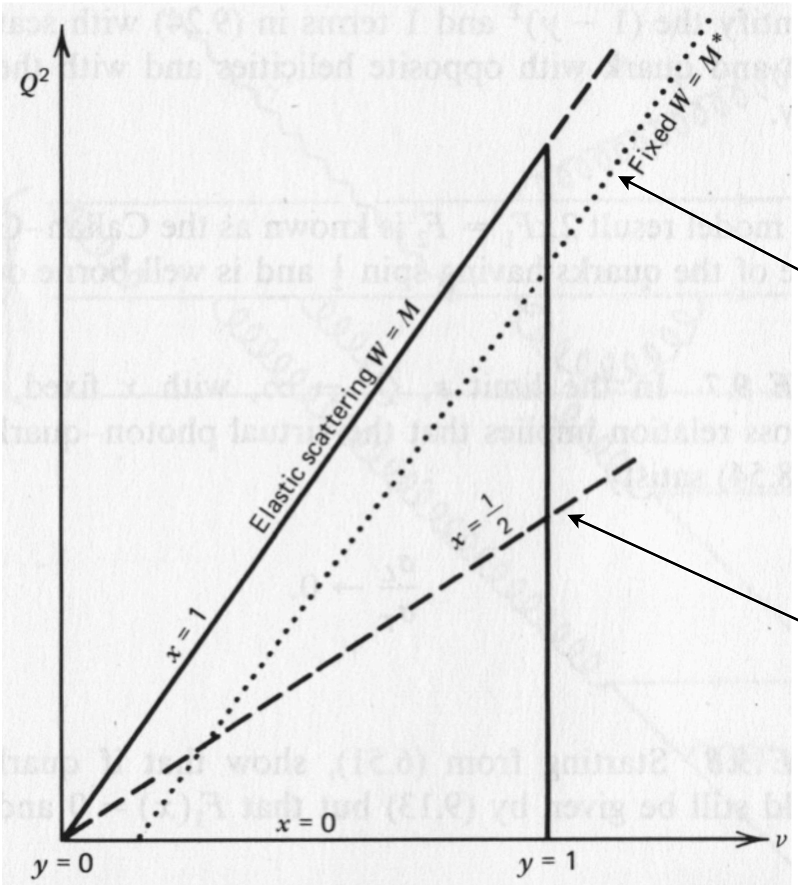
- The invariant mass of the hadronic system in the final state is

$$W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$$

Kinematic phase-space

Elastic scattering

$$x = 1 \rightarrow -q^2 = 2M\nu \rightarrow W^2 = M^2$$



Line of constant invariant mass

Line of constant momentum fraction

Cross section

- We can now use the hadronic tensor to calculate the matrix element
- In the laboratory frame:

$$(L^e)^{\mu\nu} W_{\mu\nu} = 4EE' \left[\cos^2\frac{\theta}{2} W_2(\nu, q^2) + \sin^2\frac{\theta}{2} 2W_1(\nu, q^2) \right]$$

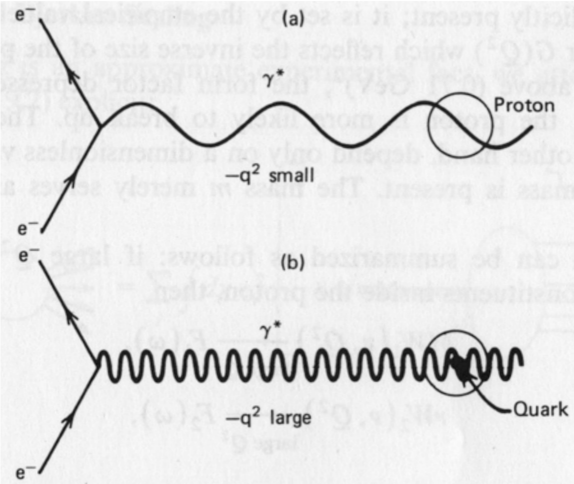
- Using the flux factor and phase-space factor

$$\frac{dq}{dE' d\Omega} = \frac{\alpha^2}{4E^2 sin^4\frac{\theta}{2}} \left[\cos^2\frac{\theta}{2} W_2(\nu, q^2) + \sin^2\frac{\theta}{2} 2W_1(\nu, q^2) \right]$$

- Integrating on the outgoing electron energy

$$\frac{dq}{d\Omega} = \frac{\alpha^2}{4E^2 sin^4\frac{\theta}{2}} \frac{E'}{E} \left[\cos^2\frac{\theta}{2} W_2(\nu, q^2) + \sin^2\frac{\theta}{2} 2W_1(\nu, q^2) \right]$$

- The key factor for understanding the proton substructure is the wavelength of the probing photon

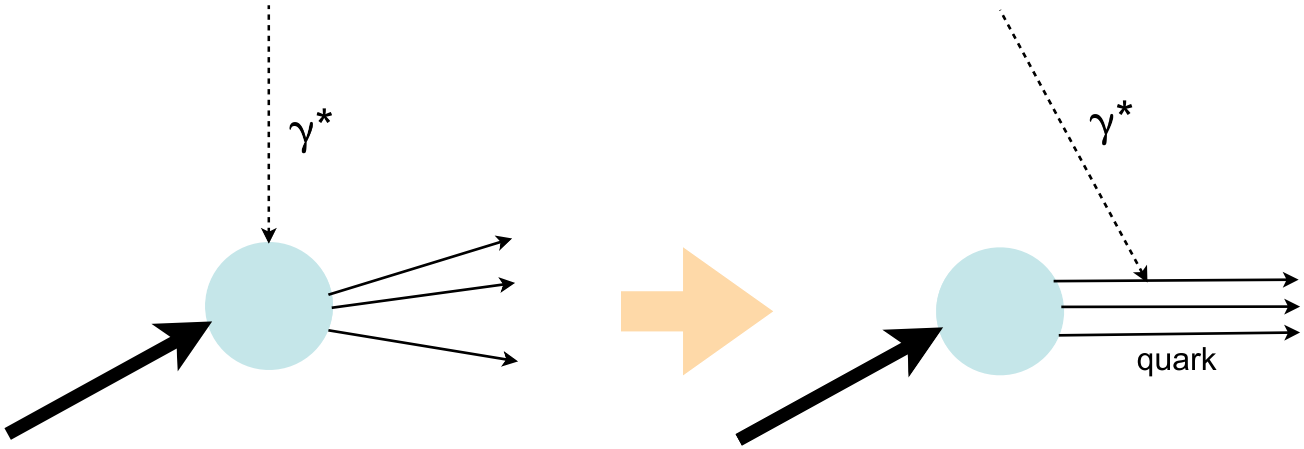


$$\lambda \sim \frac{1}{\sqrt{-q^2}} \sim 1 \text{ Fermi}$$

$$\lambda \sim \frac{1}{\sqrt{-q^2}} \ll 1 \text{ Fermi}$$

Bjorken Scaling

- In 1968 J.Bjorken proposed that in the structure functions should depend only on the **ratio** ν/q^2 (proportional to x) in the limit $q^2\to\infty$ and $\nu\to\infty$
- In other words: at large $Q^2\equiv -q^2$ the inelastic e-p scattering is viewed as elastic scattering of the electron on **free "partons" within the proton**

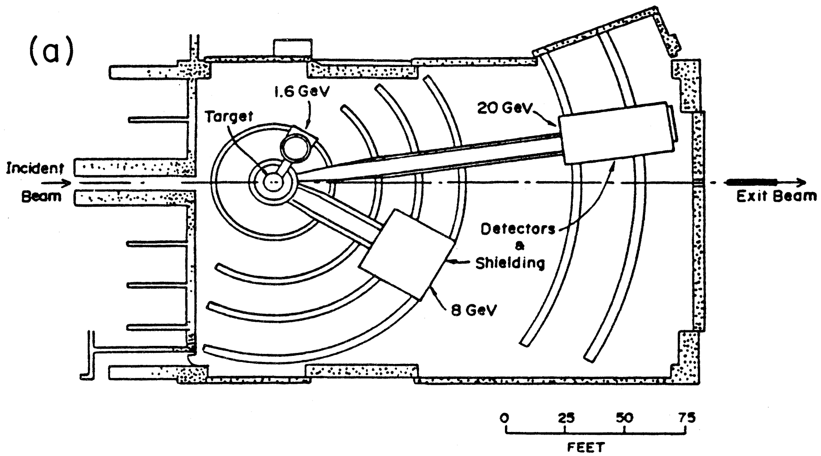


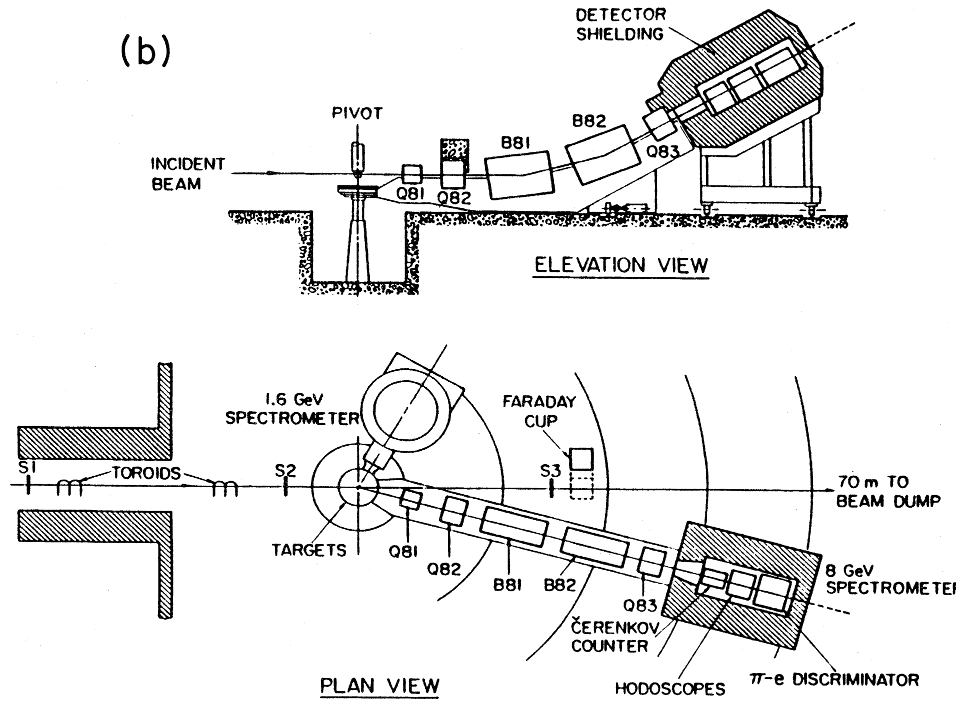
$$\lim_{Q^2\to\infty,\nu/Q^2\ \text{fixed}} \nu W_2(Q^2,\nu) = F_2(x)$$

$$\lim_{Q^2\to\infty,\nu/Q^2\ \text{fixed}} M W_1(Q^2,\nu) = F_1(x)$$

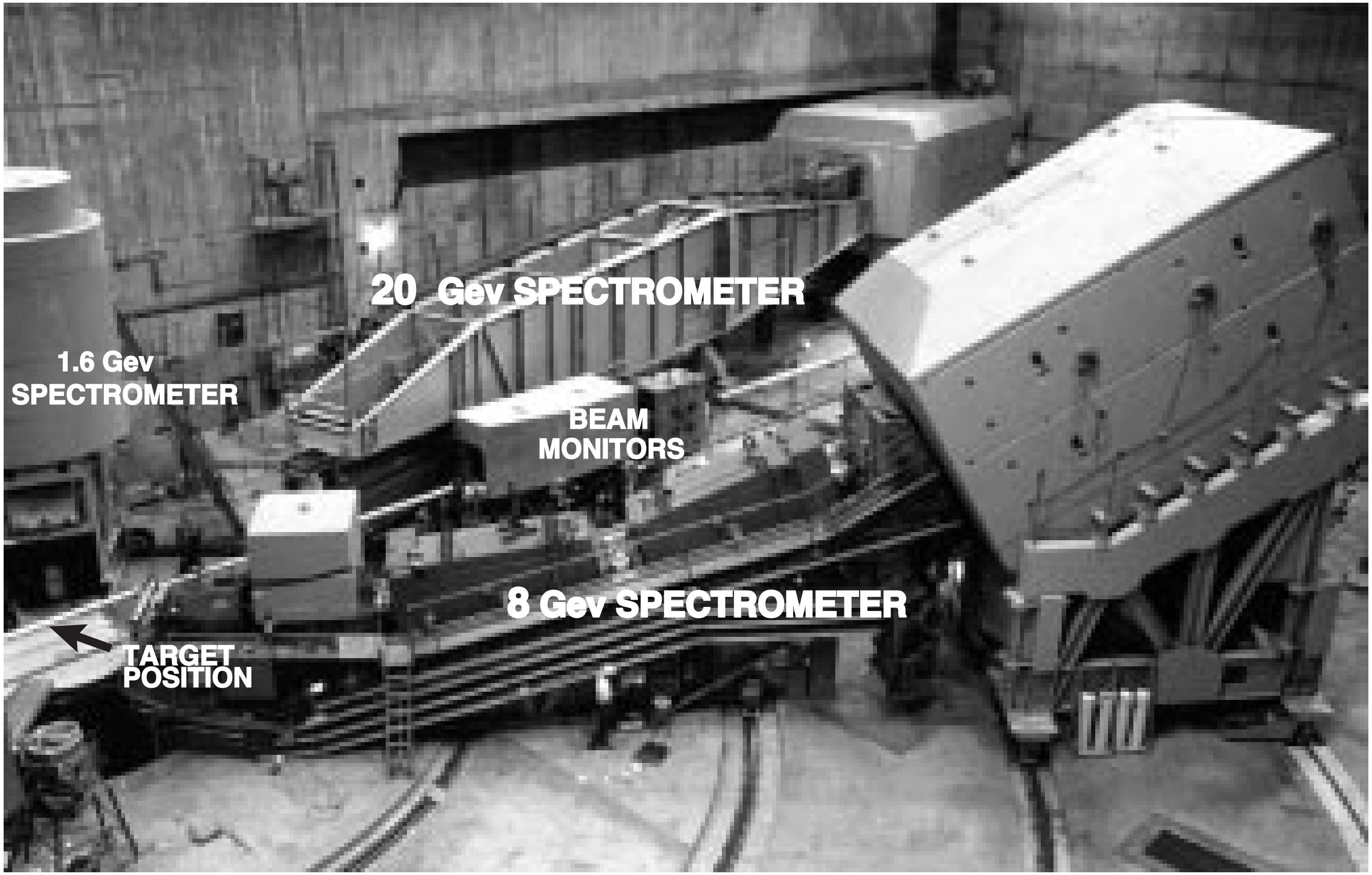
Bjorken - 1969

SLAC-MIT experiment

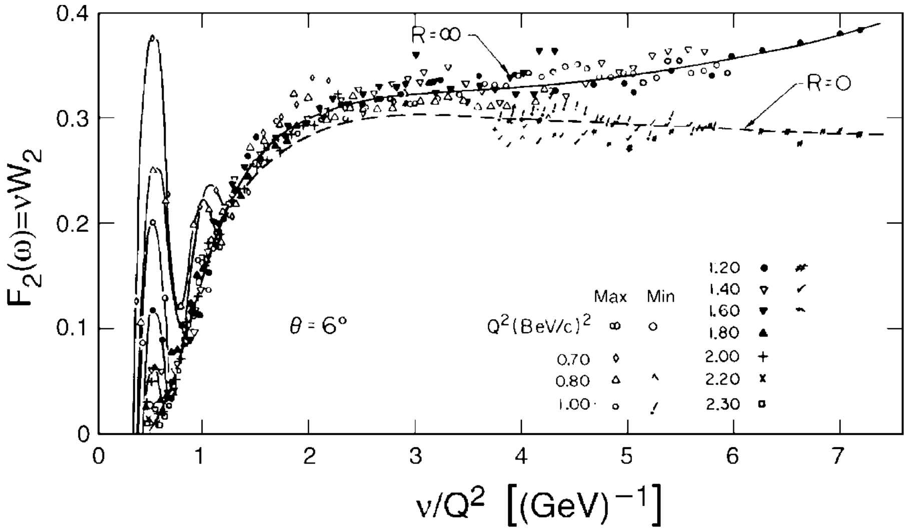




SLAC-MIT experiment



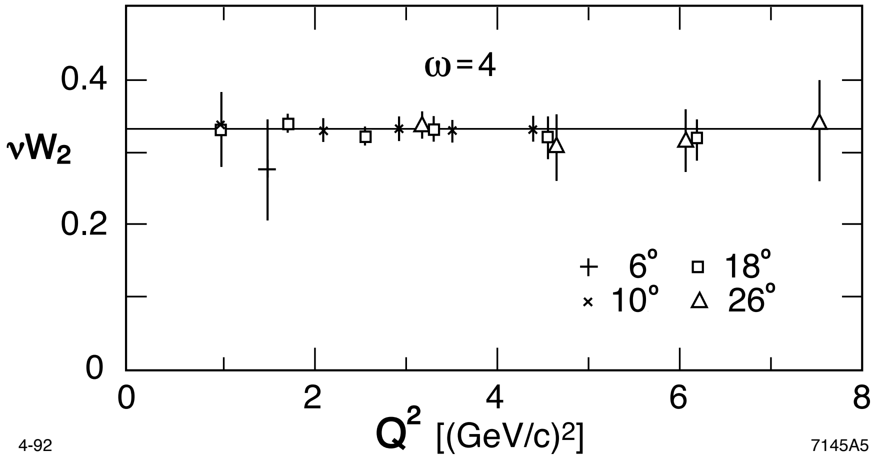
First hints of Bjorken's scaling



First data at 6°: F_2 plotted as function of the scaling variable ν/Q^2 is roughly independent on Q^2

More data at different angles:
F_2 for a fixed ω does not depend on the transferred momentum Q^2

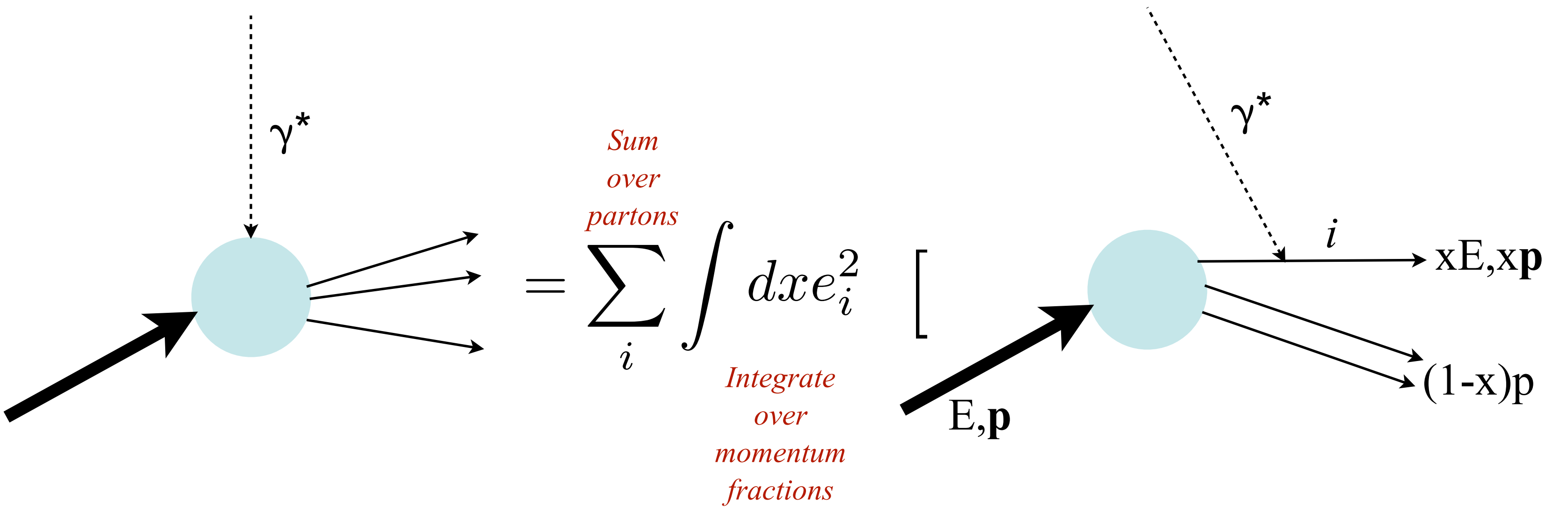
$$\omega = \frac{2q \cdot p}{Q^2} = \frac{2M\nu}{Q^2}$$



Friedman, Kendal and Taylor - 1969

Parton distributions

- Partons carry a different fraction x of the proton's momentum and energy



- The probability that the struck parton carries a fraction x of the proton momentum is usually called **parton distribution** or **parton density function**
- Total probability must be equal to one:

$$f_i(x) = \frac{dP_i}{dx}$$

$$\sum_i \int dx \; x f_i(x) = 1$$

R.Feynman - 1969

Structure function revisited

- In the Feynman's parton model the structure functions are sums of the parton densities constituting the proton

$$\nu W_2(\nu, Q^2) \rightarrow F_2(x) = \sum_i e_i^2 x f_i(x)$$

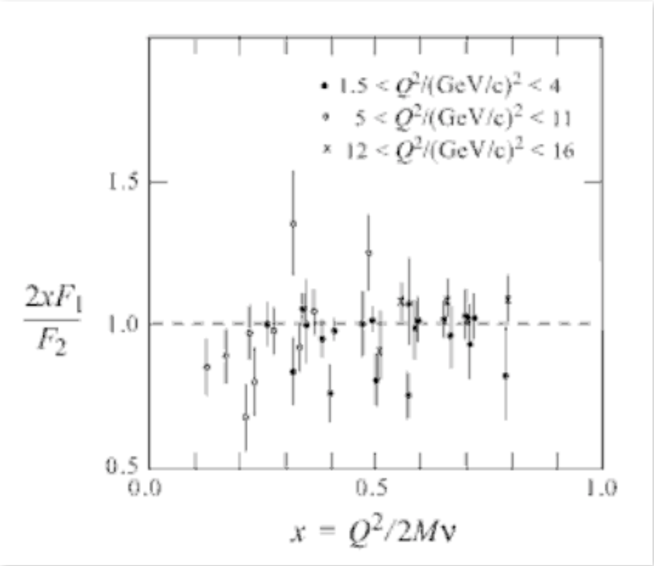
$$M W_1(\nu, Q^2) \rightarrow F_1(x) = \frac{1}{2x} F_2(x)$$

- The result $2xF_1 = F_2$ is known as **Callan-Gross** relation and is a consequence of quarks having spin 1/2
- Comparing e-p with e-μ scattering cross sections (with $m \equiv quark\ mass$):

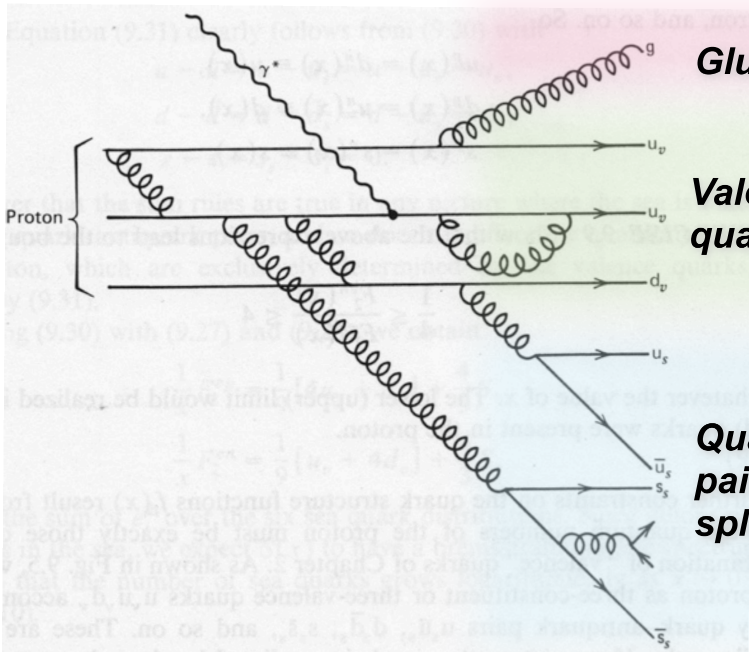
$$\frac{2W_1}{W_2} = \frac{Q^2}{2m^2}, \quad W_1 = F_1/M, \quad W_2 = F_2/\nu$$

$$[m = xM, \quad Q^2 = 2m\nu] \rightarrow \frac{F_1}{F_2} = \frac{1}{2x}$$

The parton carries a fraction x of the proton mass M



Proton/Neutron parton densities



Gluons

Valence quarks

Quark-antiquark pairs from gluon splitting ("sea")

$$\frac{1}{x}F_2^{ep} = \left(\frac{2}{3}\right)^2 [u^p + \bar{u}^p] + \left(\frac{1}{3}\right)^2 [d^p + \bar{d}^p] + \left(\frac{1}{3}\right)^2 [s^p + \bar{s}^p].$$

- We write the equivalent structure function for the **neutron** as

$$\frac{1}{x}F_2^{en} = \left(\frac{2}{3}\right)^2 [u^n + \bar{u}^n] + \left(\frac{1}{3}\right)^2 [d^n + \bar{d}^n] + \left(\frac{1}{3}\right)^2 [s^n + \bar{s}^n].$$

- Proton and neutron parton densities are correlated

$$u^p(x) = d^n(x) \equiv u(x)$$
$$d^p(x) = u^n(x) \equiv d(x)$$
$$s^p(x) = s^n(x) \equiv s(x)$$

Constraints to parton densities

- We assume the three lightest quark flavours (u,d,s) occur with equal probability in the sea

$$u_s = \bar{u}_s = d_s = d_s = s_s = s_s = S(x)$$
$$u(x) = u_v(x) + u_s(x)$$
$$d(x) = d_v(x) + d_s(x)$$

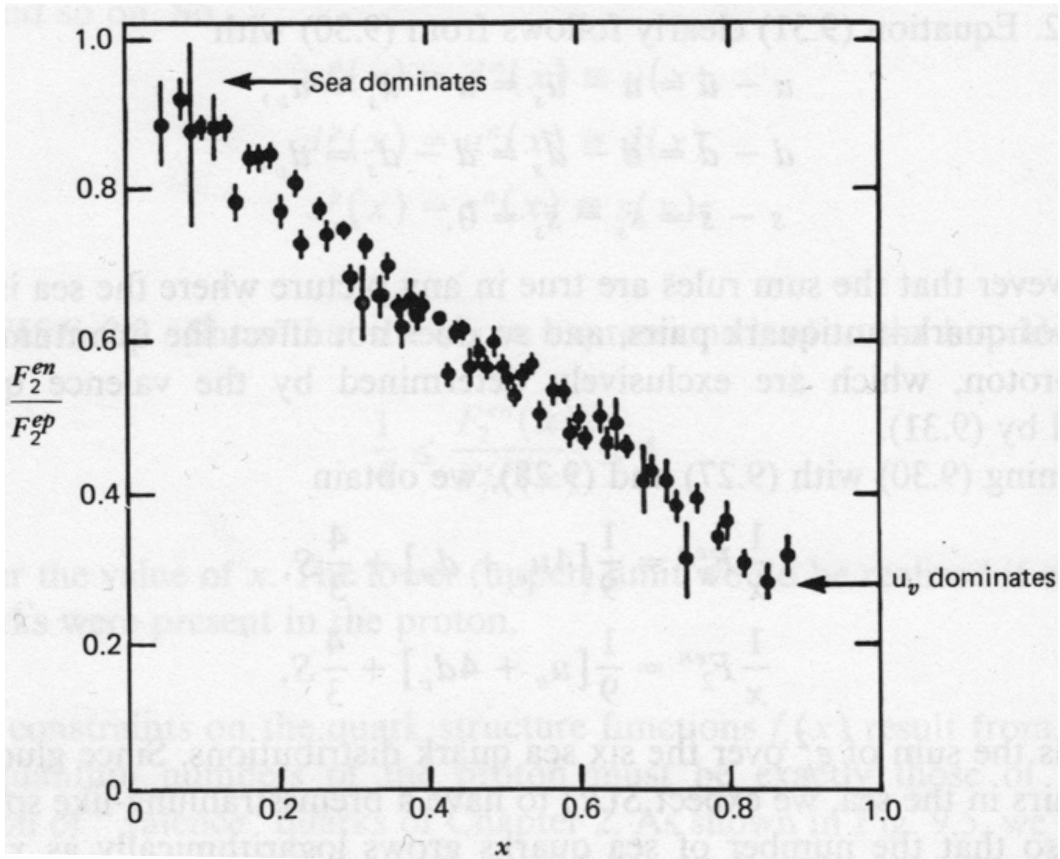
- Combining all constraints:

$$\frac{1}{x}F_2^{ep} = \frac{1}{9}[4u_v + d_v] + \frac{4}{3}S$$
$$\frac{1}{x}F_2^{en} = \frac{1}{9}[u_v + 4d_v] + \frac{4}{3}S$$

- At small momenta (x~0) the structure function is dominated by **low-momentum quark** pairs constituting the “sea”. For x~1 the **valence quarks** dominate and the ratio F_2^{en}/F_2^{ep} becomes

$$\frac{u_v + 4d_v}{4u_v + d_v} \sim \frac{1}{4}$$

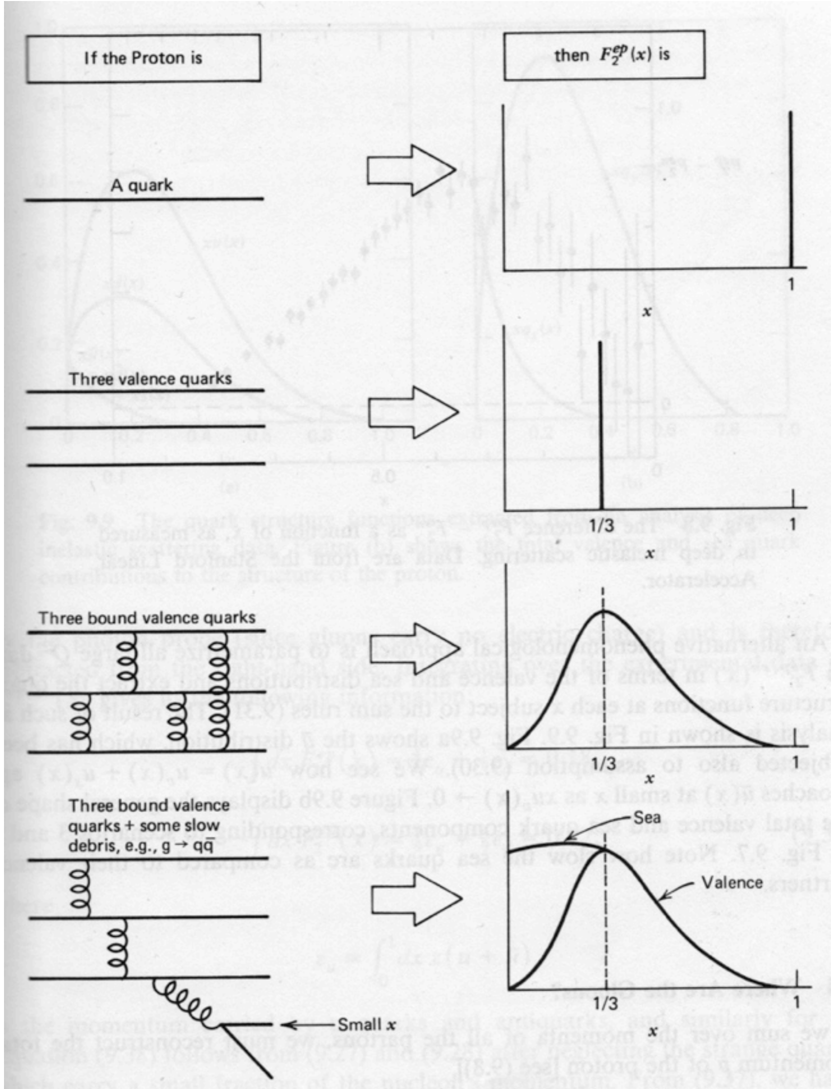
Ratio of structure functions



low momentum fractions (sea)

Partons carry most of the hadron momentum (valence)

Summary of F_2 proton



How about gluons?

- Summing over the momenta of all partons we should reconstruct the total proton momentum:

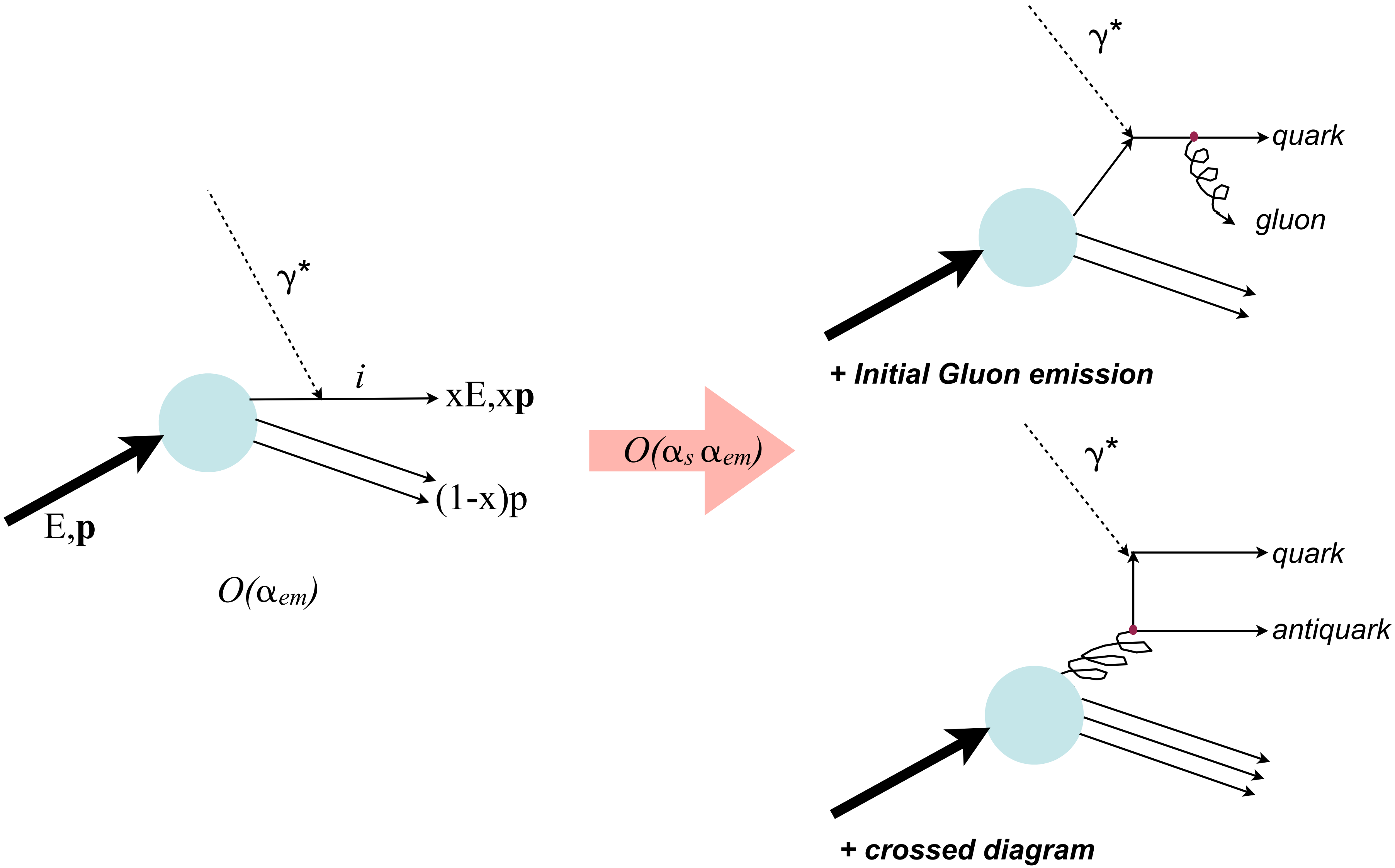
$$\int_0^1 dx\; x(u + \bar{u} + d + \bar{d} + s + \bar{s}) = 1 - \frac{p_g}{p} = 1 - \epsilon_g$$

- Neglecting the small fraction carried by the strange quarks we have and using the results of experimental data

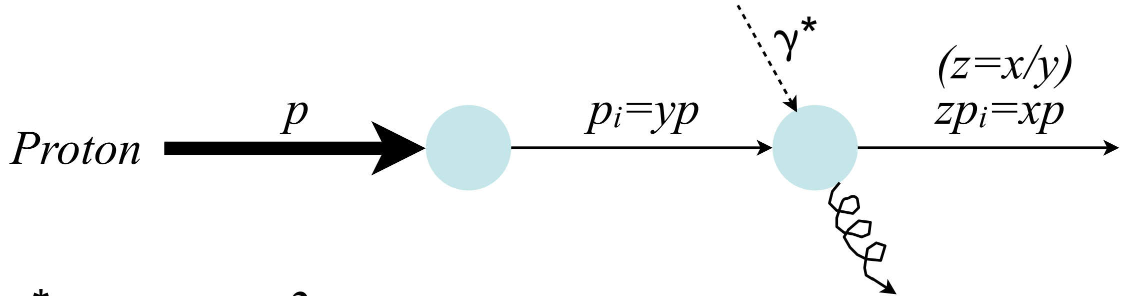
$$\epsilon_u \equiv \int_0^1 dx\; x(u + \bar{u}) \quad \Rightarrow \quad \epsilon_g \simeq 1 - \epsilon_u - \epsilon_d = 1 - 0.36 - 0.18 = 0.46$$

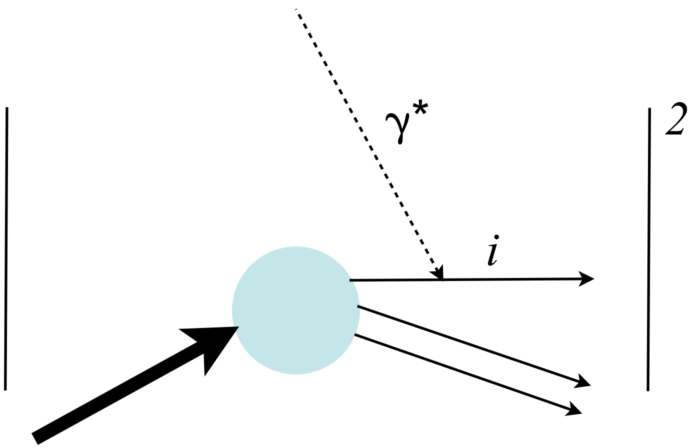
Experimental data indicate that about 50% of the proton momentum is carried by neutral partons, not by quarks!

Gluons and the parton model



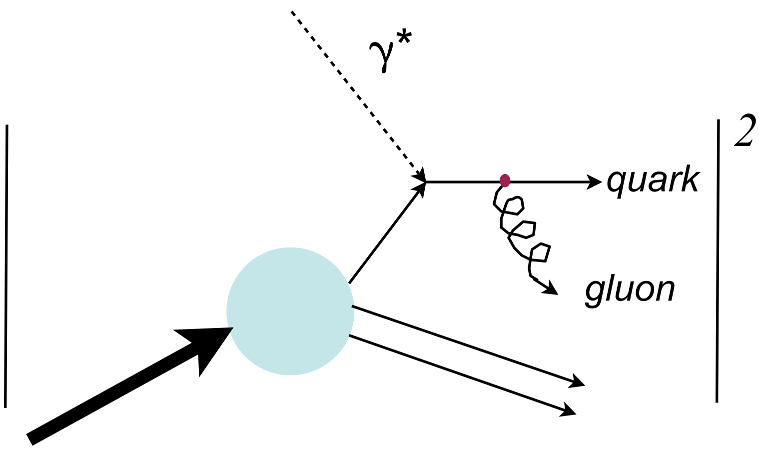
Gluon emission: contribution to F_2





Scaling: no Q2 dependence

$$\frac{F_2(x,Q^2)}{x} = \sum_i e_i^2 \int_x^1 \frac{dy}{y} f_i(y)\delta(1-x/y) = \sum_i e_i^2 f_i(x)$$



+ ***Initial Gluon emission***

Scaling violation!

$$\frac{F_2^{\gamma^* q\to qg}(x,Q^2)}{x} = \sum_i e_i^2 \int_x^1 \frac{dy}{y} f_i(y) \left[\frac{\alpha_s}{2\pi} P_{qq}(x/y)\log\frac{Q^2}{\mu^2}\right]$$

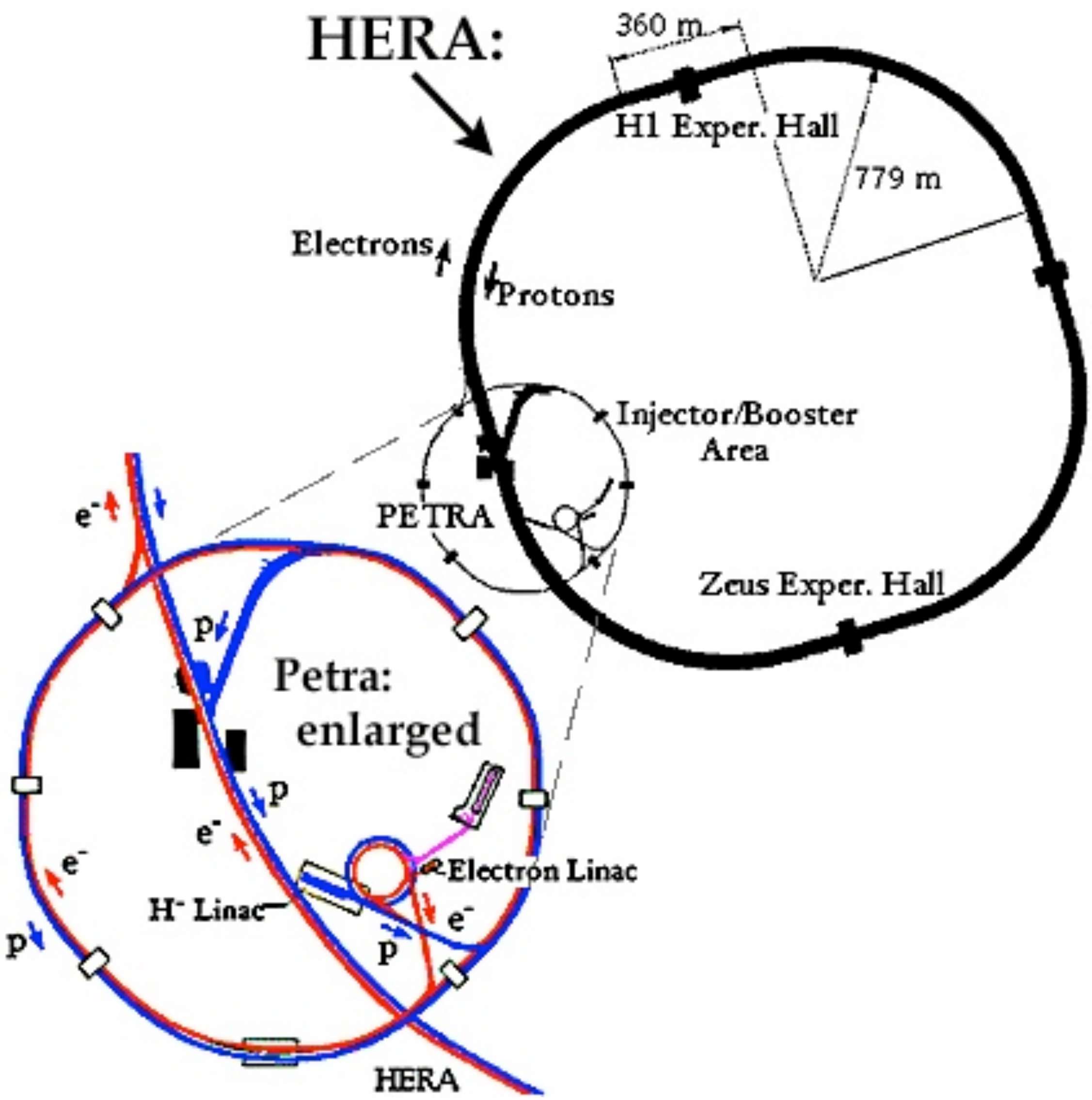
Cut-off for soft gluons

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

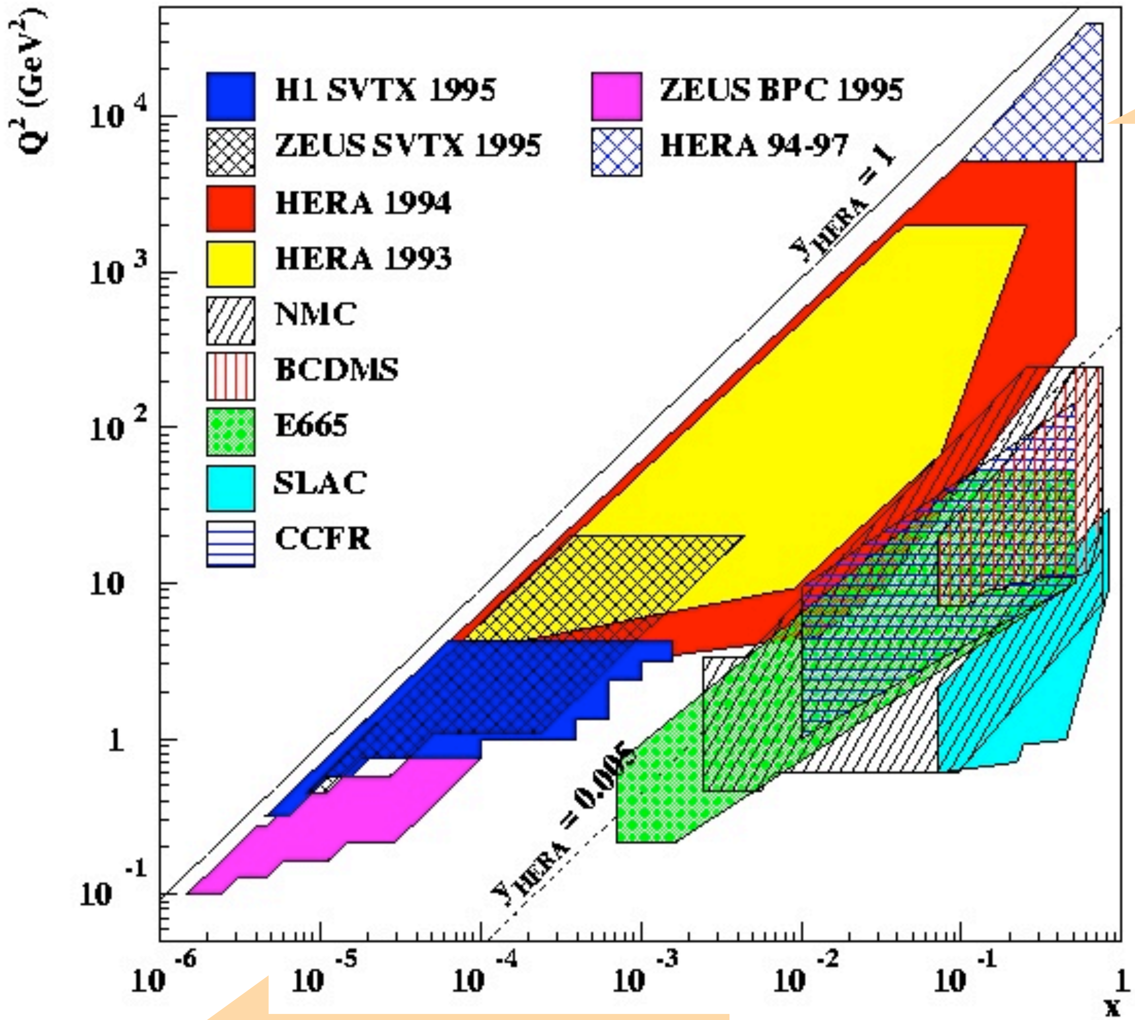
Splitting function:
probability of a quark to emit a gluon and reduce momentum by a fraction z
N.B.: divergent for soft gluons ($z\to 1$)

Experimental techniques

HERA accelerator complex



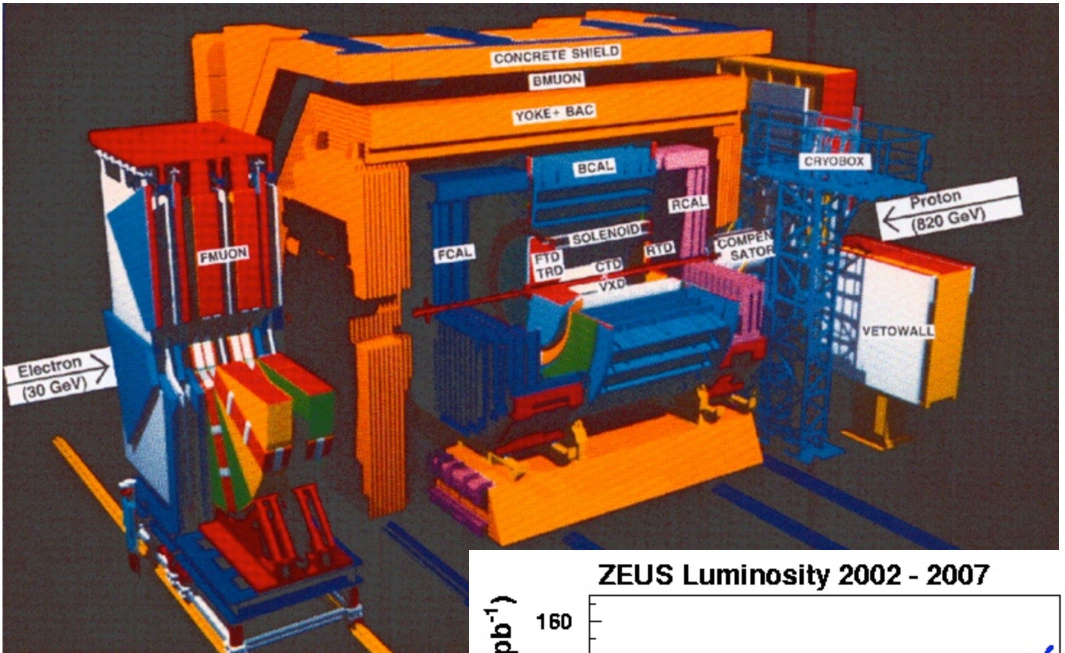
Kinematic region

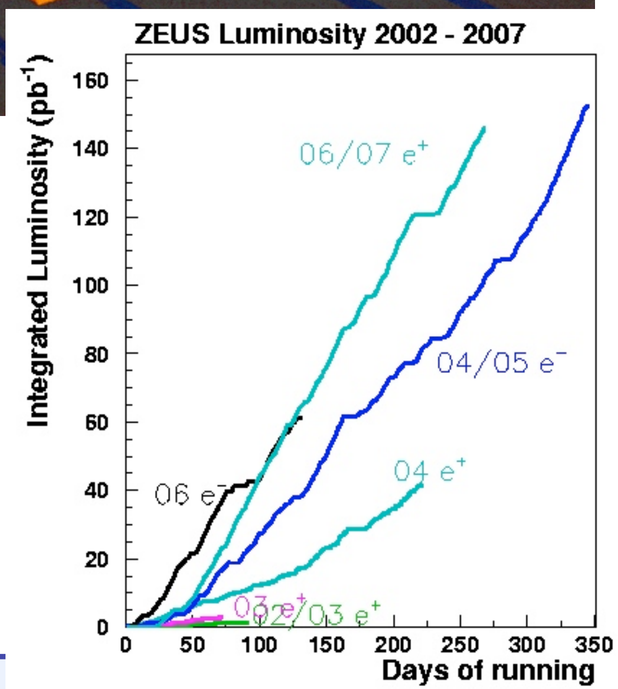


Larger momentum transfers (large electron scattering angles)

Larger sensitivity to sea quarks (gluons)

HERA experiments: ZEUS





Subdetectors:

1) Central tracker
- electron momentum
- charged particles in jet
- muon momentum

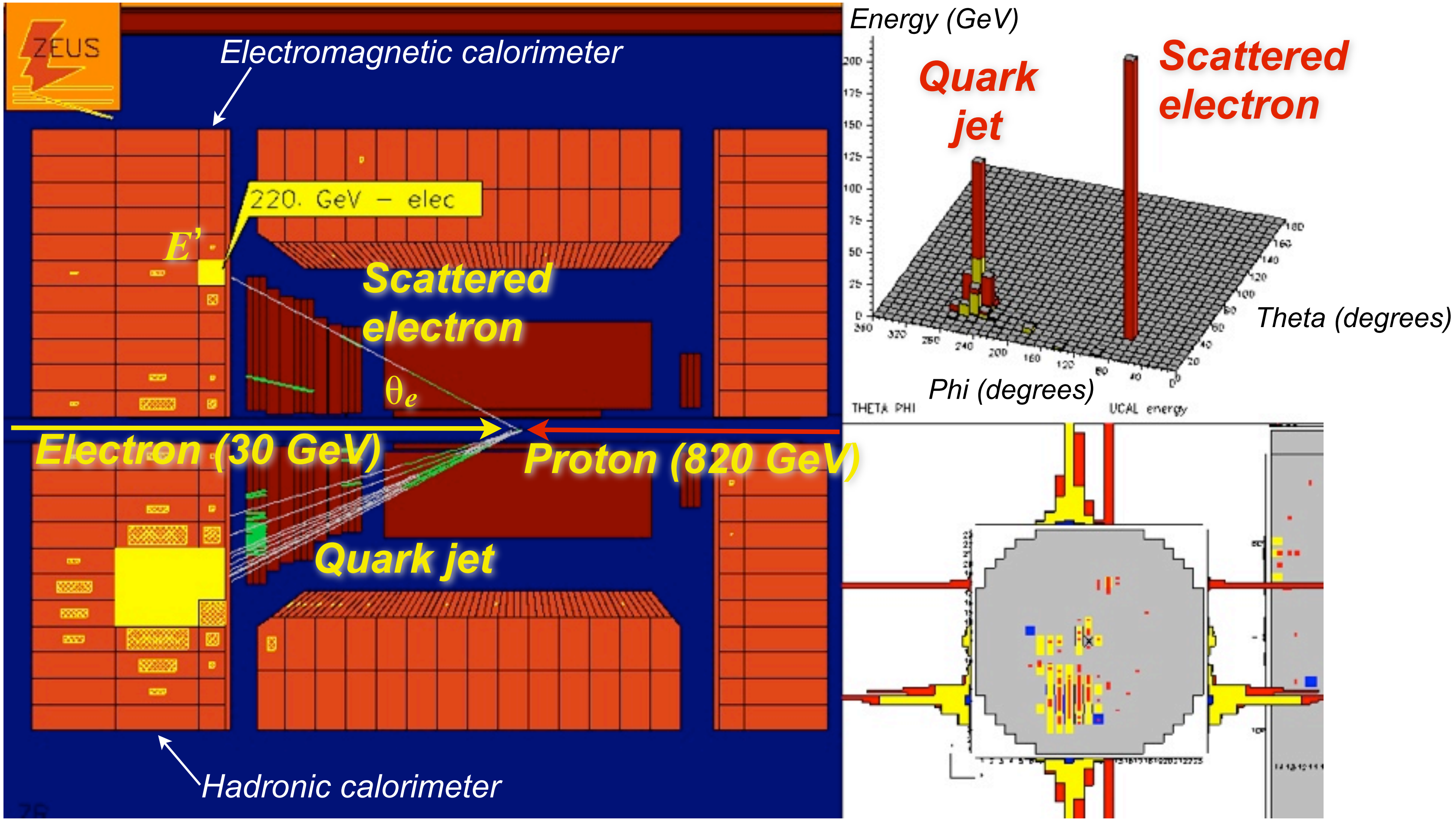
2) Electromagnetic calorimeter
- electron (and photon) energy

3) Hadronic calorimeter
- jet energy

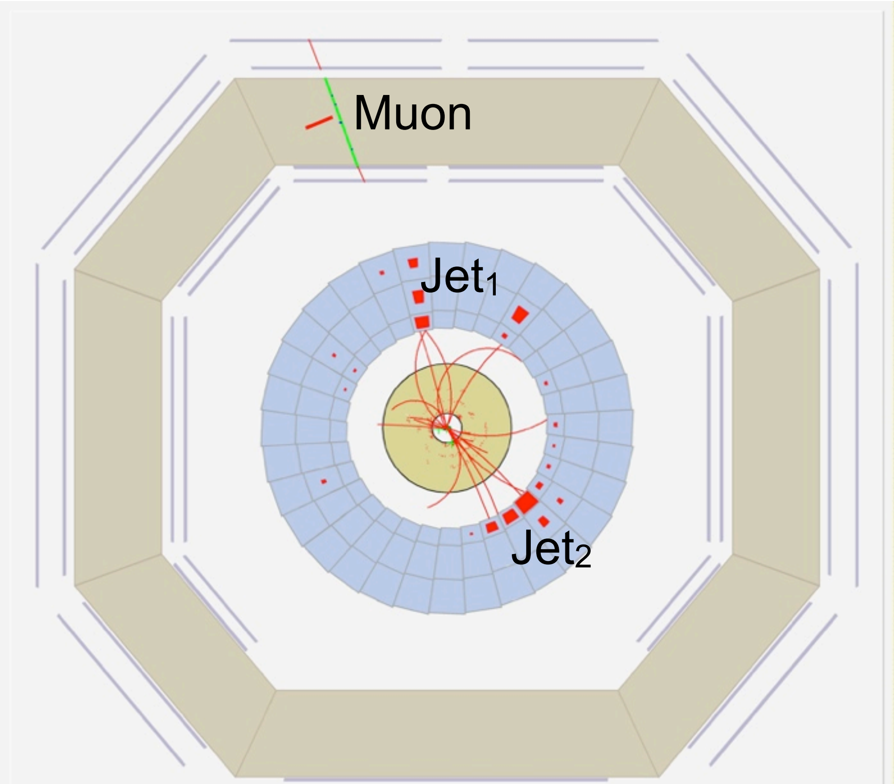
4) Muon detectors:
- muon ID and momentum

Deep Inelastic Scattering event

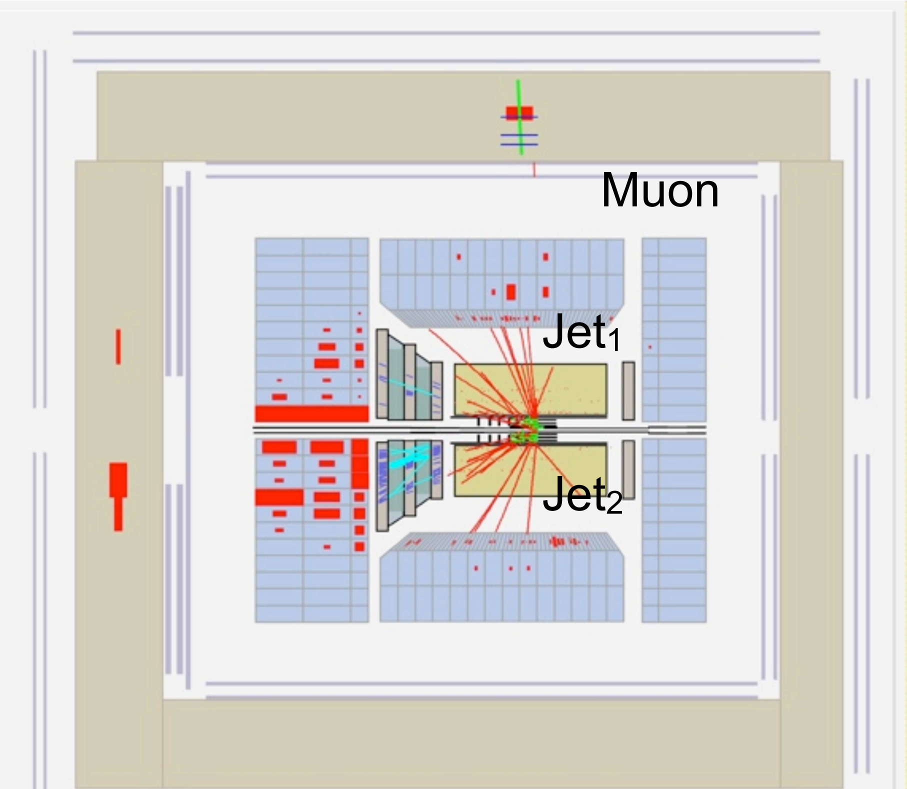
e p →e jet X



Events with two jets



Transverse view



Longitudinal view

$e\ p \rightarrow e\ jet_1\ jet_2\ X$

partonic subprocess: $\gamma^* g \rightarrow q$ anti-q

Only possible with gluons!

Kinematic reconstruction

- To fully characterize a deep inelastic scattering event kinematics both Q^2 and x (or y) have to be measured

$$e(k) + p(P) \rightarrow e(k') + X$$

$$Q^2 = -q^2 = -(k-k')^2,$$
$$x = \frac{Q^2}{2P \cdot q},$$
$$y = Q^2/(sx)$$

- Both variables can be measured e.g. detecting only the scattered electron

$$y_e = 1 - \frac{E'_e}{2E_e}(1 - \cos\theta_e),$$
$$Q^2_e = 2E_e E'_e(1 + \cos\theta_e).$$

- More precise methods combine the measurements (energy and polar angle) of both electron and hadronic system

Measurement of F_2 proton

- The measurement of F_2 is given by the double differential e-p cross section as function of x and Q^2:

Measurement

$$\frac{d\sigma^2_{exp}}{dxdQ^2} = \frac{N(x,Q^2)}{\epsilon \Delta x \Delta Q^2 L}$$

Number of detected events in a x,Q2 bin

Detector efficiency correction (from Monte Carlo simulation)

Bin size & integrated luminosity

Theory

$$\frac{d\sigma^2}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^2}[1+(1-y)^2]F_2(x,Q^2)\delta$$

$$\delta = (1-\delta_L-\delta_3)(1+\delta_r)$$

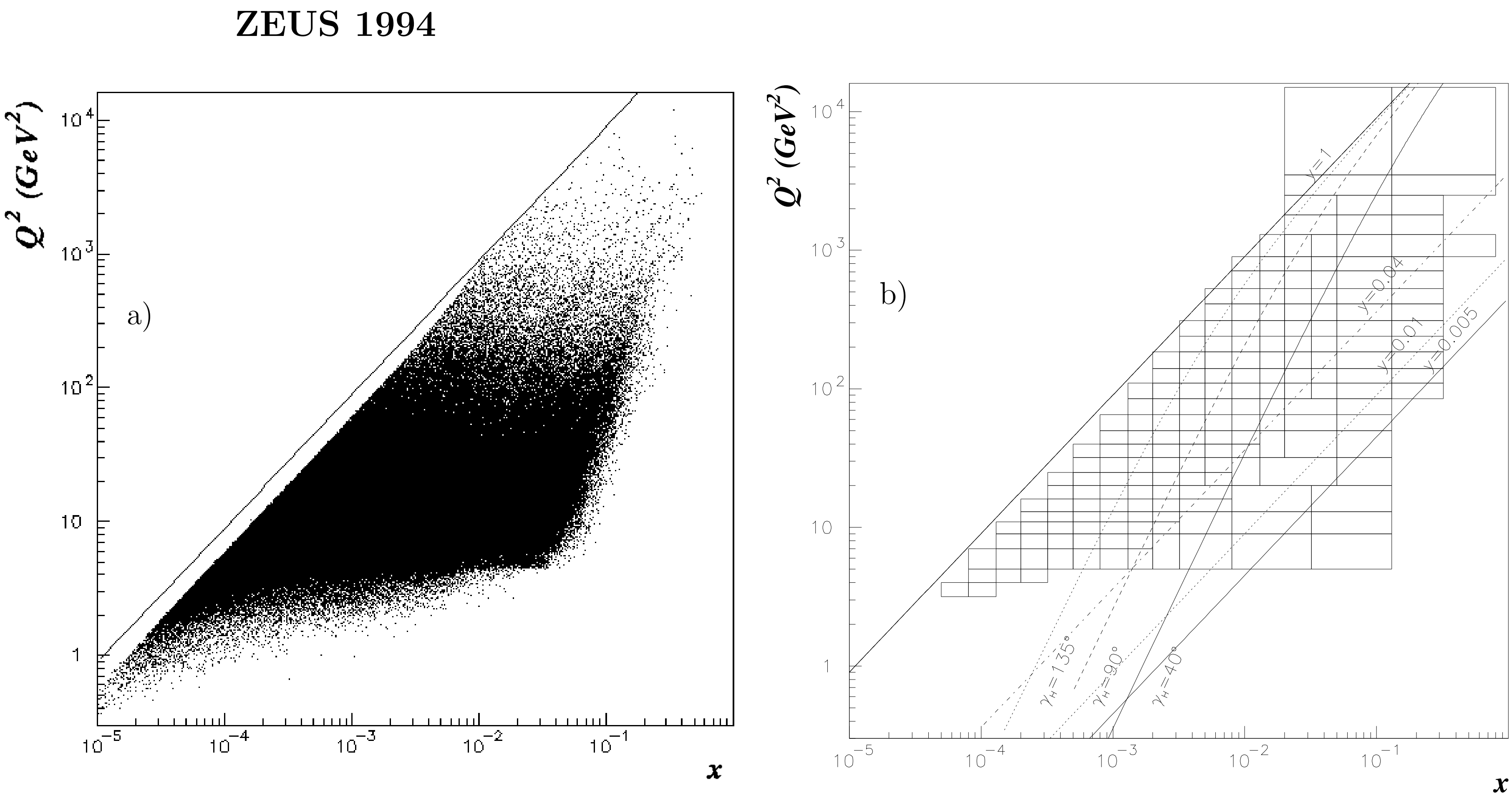
to first order can be neglected

Longitudinal structure function

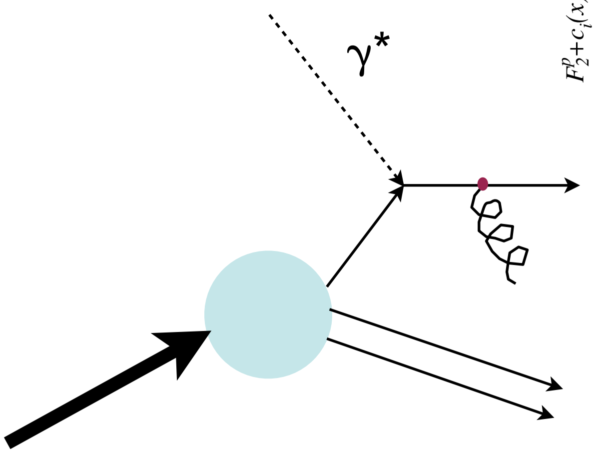
Parity-violating term due to Z0 exchange

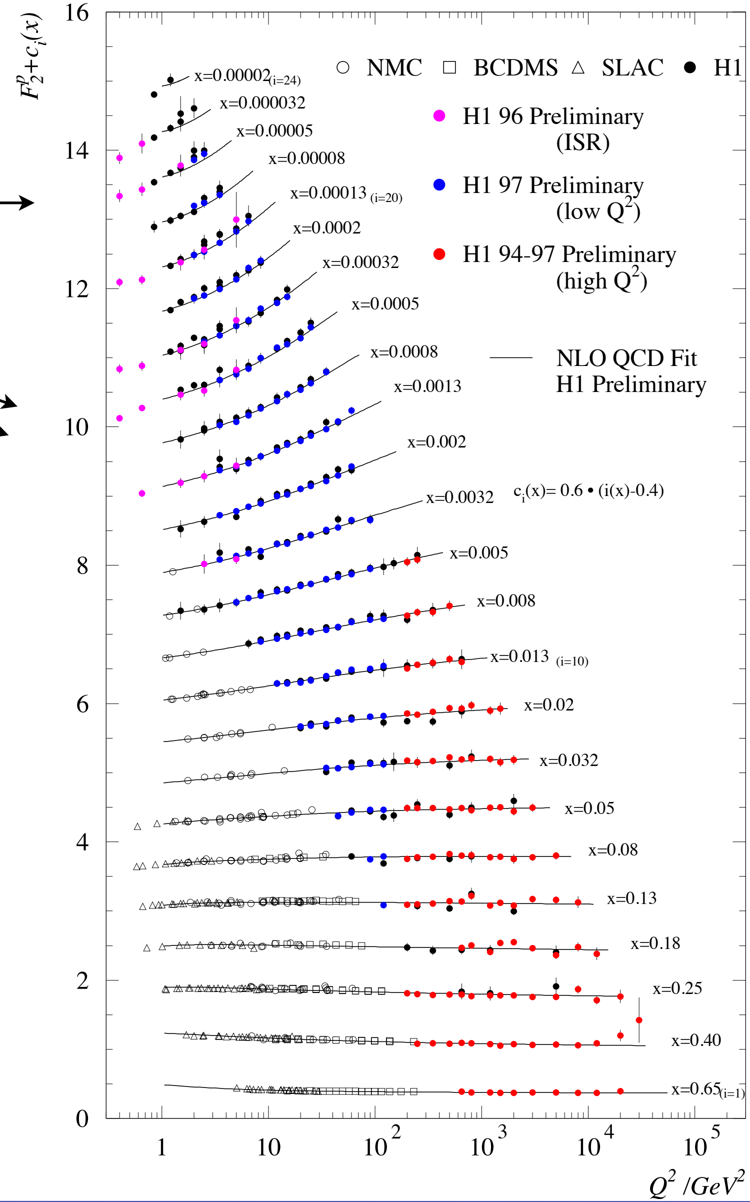
Electroweak radiative correction

Measured kinematic range

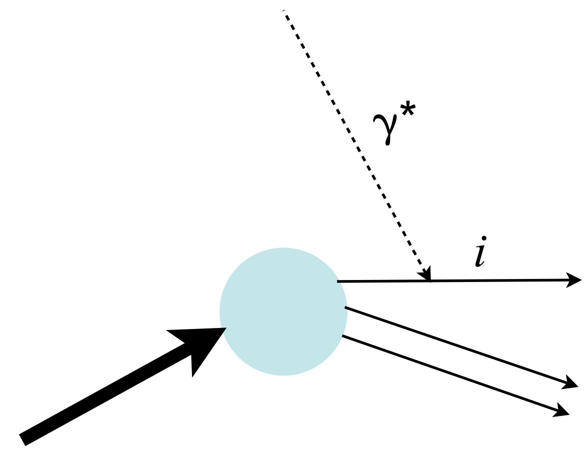


F_2 results from HERA



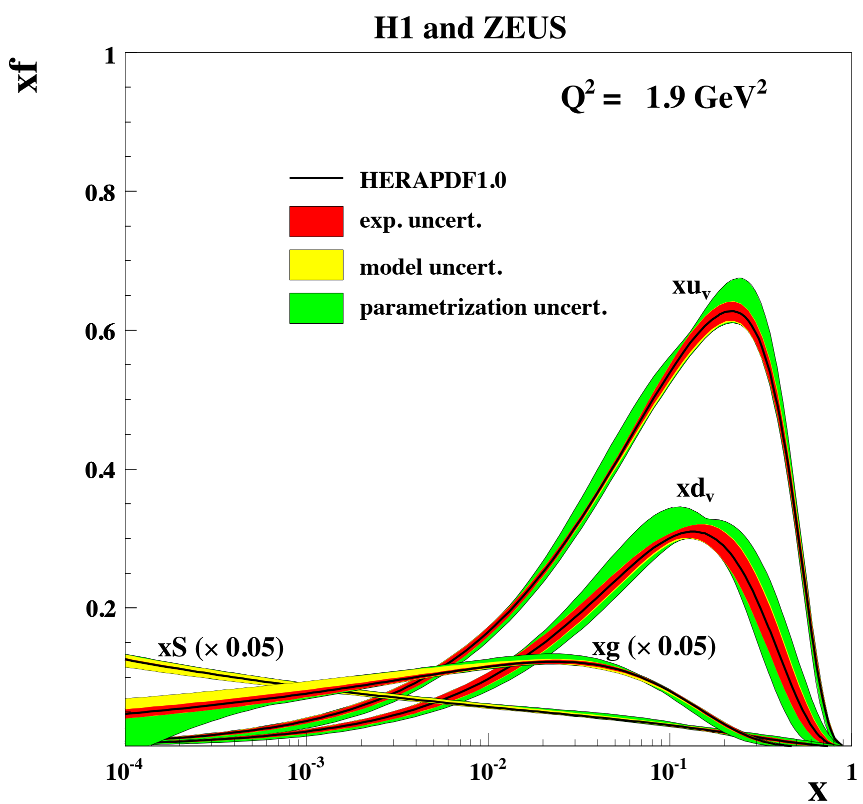


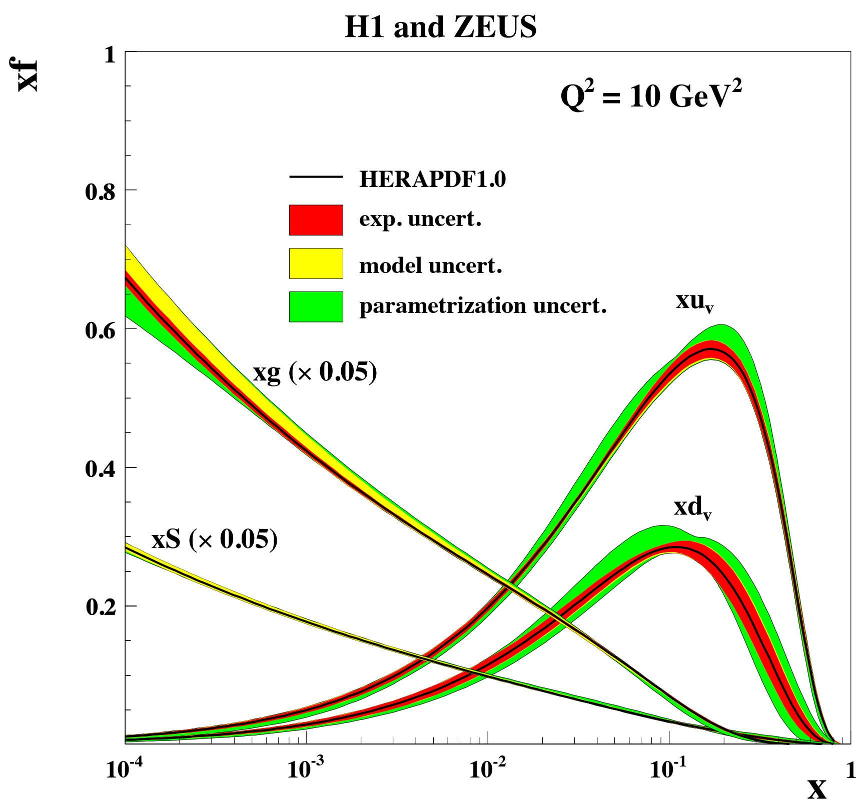
Scaling violation (logarithmic dependence) $x<0.01$



Approximate scaling, $x \sim 10^{-1}$

Parton distributions from HERA





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