Particle Physics Phenomenology II

FS 11, Series 12

Due date: 30.05.2011, 1 pm

Exercise 1

In this exercise You will compute the initial state collinear singularities encountered at the next to leading order in QCD, and show that these can be renormalised into the parton distributtion function. We will look at the process $b\bar{b} \to H$.

i) Show that at leading order the cross section for $b\bar{b} \to H$ is

$$\sigma_B = \frac{\pi y_b^2}{\hat{s}} \frac{\delta(1-z)}{6}$$

where $z = m_h^2/\hat{s}$.

ii) Now turn to the real corrections, these come from the process $b\bar{b} \to Hg$. When one attempts to integrate the squared real matrix element over the full $2 \to 2$ phase space, one encounters singularities. We will regulate these using dimensional regularisation. Given that in $d = 4 - 2\epsilon$ dimensions the phasespace measure is

$$\Phi_2(s; m_h, 0) = \frac{1-z}{\Gamma(1-\epsilon)} \frac{(4\pi)^\epsilon}{8\pi} \int_0^1 d\lambda \left(\hat{s}(1-z)^2 \lambda(1-\lambda)\right)^{-\epsilon}.$$

such that the Mandelstam variables are

$$\begin{aligned} s_{13} &= -\lambda(\hat{s} - m_h^2) \\ s_{23} &= -(1-\lambda)(\hat{s} - m_h^2) \end{aligned}$$

and that the squared averaged amplitude for the real correction is

$$|M|^2 = \alpha_s(\mu) y_b(\mu) \frac{16\pi}{9} \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \frac{(\hat{s}^2 + m_h^4) - \epsilon(\hat{s} - m_h^2)^2}{s_{13} s_{23}}$$

show that the contribution of the real correction to the cross section is

$$\begin{aligned} \sigma_{Real} = \; & \frac{\alpha(\mu) y_b^2}{\hat{s}} \left[\frac{1}{\epsilon^2} \frac{2}{9} \delta(1-z) + \frac{1}{\epsilon} \left(\frac{2}{9} \delta(1-z) \ln \frac{\mu^2}{s} - \frac{2}{9} \frac{1+z^2}{(1-z)_+} \right) \right. \\ & - \frac{1}{18} \left(\pi^2 - 2 \ln^2 \frac{\mu^2}{s} \right) \delta(1-z) + \frac{2}{9}(1-z) - \frac{2}{9} \frac{1+z^2}{(1-z)_+} \ln \frac{\mu^2}{s} \\ & \left. + 4(1+z^2) \left[\frac{\ln(1-z)}{1-z} \right]_+ \right]. \end{aligned}$$

Hint: You will need to use the following identity

$$(1-z)^{-1+\epsilon} = \frac{\delta(1-z)}{\epsilon} + \sum_{n=0}^{\infty} \frac{\epsilon^n}{n!} \left[\frac{\log^n(1-z)}{1-z} \right]_+$$

with the plus-distribution defined by

$$\int_0^1 dx f(x) \left[\frac{g(x)}{x}\right]_+ = \int_0^1 dx g(x) \left[\frac{f(x)-f(0)}{x}\right].$$

iii) Given that the renormalised virtual contribution is given by

$$\sigma_V = \Bigg[-2/9 \left(\frac{\mu^2}{s}\right)^{\epsilon} {y_b}^2 \alpha_s e^{\gamma_E\,\epsilon} \cos(\pi\,\epsilon) \left(\frac{\Gamma(1+\epsilon)(\Gamma(-\epsilon))^2}{\Gamma(1-2\,\epsilon)} + \epsilon\,\Gamma(\epsilon)\,B(1-\epsilon, 1-\epsilon)\right)$$
$$-\frac{\alpha y_b^2}{3m_h^2}\frac{1}{\epsilon}\Bigg]\frac{\delta(1-z)}{m_h^2}$$

compute the NLO QCD correction. Show in particular that the double pole $1/\epsilon^2$ cancels, but that the coefficient of $1/\epsilon$ comes with the following coefficient

$$\sigma_C = \frac{\alpha_s y_b^2}{3\epsilon\hat{s}} P_{qq}^{(0)}(z)$$

where the leading order quark-quark splitting function is defined as

$$P_{qq}^{(0)}(z) = \delta(1-z) + \frac{2}{3}\frac{1+z^2}{(1-z)_+}.$$

iv) Identify σ_C as the collinear counter term, arising from the collinear singularities absorbed in the parton distribution function (pdf) and show that the following redefinition of the pdf

$$f_i(x) \to \sum_j f_j \otimes \Gamma_{ij}(x)$$

where

$$\Gamma_{ij}(x) = \delta_{ij}\delta(1-x) - \frac{\alpha_s}{\pi}\frac{P_{ij}(x)}{\epsilon} + ..$$

will render the hadronic cross section finite. Recall that the convolution integral is defined as

$$f_1 \otimes f_2(z) = \int_0^1 dx dy f_1(x) f_2(y) \delta(xy - z).$$