Particle Physics Phenomenology II

FS 11, Series 2

Due date: 07.03.2011, 1 pm

Exercise 1 The aim of this exercise is to compute the $\mu(p_0) \to e(p_3)\bar{\nu}_e(p_1)\nu_\mu(p_2)$ decay rate using Fermi's Theory. The Fermi Lagrangian is

$$\mathcal{L}_F = -2\sqrt{2}G_F \left[\bar{\psi}_{\nu_\mu}\gamma^\rho P_L \psi_\mu\right] \left[\bar{\psi}_e \gamma_\rho P_L \psi_{\nu_e}\right].$$

i) Prove the Fierz rearrangement

$$(\bar{u}_{1L}\bar{\sigma}^\mu u_{2L})(\bar{u}_{3L}\bar{\sigma}_\mu u_{4L}) = -(\bar{u}_{1L}\bar{\sigma}^\mu u_{4L})(\bar{u}_{3L}\bar{\sigma}_\mu u_{2L}).$$

to show that

$$\mathcal{L}_F = 2\sqrt{2}G_F \left[\bar{\psi}_{\nu_\mu}\gamma^\rho P_L \psi_{\nu_e}\right] \left[\bar{\psi}_e \gamma_\rho P_L \psi_\mu\right].$$

Hint: Work in the Weyl representation and make use of $(\sigma^\mu)_{\alpha\beta}(\sigma_\mu)_{\gamma\delta} = 2\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}$.

ii) Show that the squared (spin summed and averaged) amplitude for this process can hence be written as

$$|M|^2 = \frac{1}{2}(2\sqrt{2}G_F)^2 Tr(\not{p}_3\gamma^\mu P_L(\not{p}_0 + m_\mu)P_R\gamma^\rho)Tr(\not{p}_1\gamma_\mu\not{p}_2\gamma_\rho P_L).$$

iii) Recall that the differential decay rate is

$$d\Gamma = \frac{(2\pi)^4}{2E_\mu} dR_3(p_0; p_1, p_2, p_3)|M|^2$$

where $dR_3(p_0; p_1, p_2, p_3)$ is the 3 particle phase-space element. Derive the following phase space factotrisation property

$$dR_3(p_0; p_1, p_2, p_3) = (2\pi)^3 \int ds_{12} dR_2(p_0; p_{12}, p_3) dR_2(p_{12}; p_1, p_2)$$

where $s_{12} = (p_1 + p_2)^2$ and use it to integrate out the momenta of the neutrinos p_1 and p_2. In other words show that

$$T^{\mu\rho} = \int dR_2(p_{12}; p_1, p_2) Tr(\not{p}_1\gamma_\mu\not{p}_2\gamma_\rho P_L) = -\left(\frac{s_{12}}{6(2\pi)^5}\right)\left[g^{\mu\rho} - \frac{p_{12}^\mu p_{12}^\rho}{s_{12}}\right].$$

iv) Parameterise the remaining integral in terms of the electron energy and show that

$$\int ds_{12} dR_2(p_0; p_{12}, p_3) = \int \frac{dE_3 E_3}{(2\pi)^5}.$$

and

$$\begin{aligned} s_{12} &= m_\mu(m_\mu - 2E_3) \\ p_0.p_3 &= m_\mu E_3 \\ p_0.p_{12} &= m_\mu(m_\mu - E_3) \\ p_3.p_{12} &= m_\mu E_3. \end{aligned}$$

v) Hence compute the Electron Energy spectrum

$$\frac{d\Gamma}{dE_3} = \frac{G_F^2 m_\mu^2 E_3^2}{12\pi^3}\left(3 - \frac{4E_3}{m_\mu}\right)$$

and the total decay rate

$$\Gamma = \frac{m_\mu^5 G_F^2}{192\pi^3}.$$