Particle Physics Phenomenology II

FS 11, Series 6

Due date: 04.04.2011, 1 pm

Exercise 1 This exercise concerns the unitarity bound on the mass of the Higgs boson from *longitudinal* W-W scattering.

i) Draw all (5) diagrams which contribute to the scattering amplitude for the process $W^+(p_+)W^-(p_-) \to W^+(q_+)W^-(q_-)$ in the higgsless electroweak theory.

ii) Show that in the center of mass frame of the incoming W's one can parametrize the momenta as

$$p_\pm = (E, 0, 0, \pm p), \quad q_\pm = (E, 0, \pm p \sin\theta, \pm p\cos\theta)$$

and their polarization vectors as

$$\epsilon_L(p_\pm) = \left(\frac{p}{M_W}, 0, 0, \pm\frac{E}{M_W}\right), \quad \epsilon_L(q_\pm) = \left(\frac{p}{M_W}, 0, \pm\frac{E}{M_W}\sin\theta, \pm\frac{E}{M_W}\cos\theta\right)$$

such that the polarization vectors satisfy the Lorentz condition $\epsilon(k).k = 0$ and are normalized so that $\epsilon^2 = -1$.

iii) *Optional:*
Compute the amplitude corresponding to the diagrams you drew in i) in the high energy limit, $p^2 \gg M_W^2$, to derive

$$T^1_{W^+W^-\to W^+W^-} = g_W^2\left(\frac{p^2}{M_W^2}\right)\left[\frac{\cos\theta}{2} + \frac{1}{2}\right] + \mathcal{O}\left(\frac{p^2}{M_W^2}\right)^0.$$

iv) Comment on the unitarity of $M^1_{W^+W^-\to W^+W^-}$. Show that the 2 diagrams involving the Higgs boson may for $p^2 \gg M_W^2$ be written as

$$T^2_{W^+W^-\to W^+W^-} = g_W^2\left\{\left(\frac{p^2}{M_W^2}\right)\left[-\frac{\cos\theta}{2} - \frac{1}{2}\right] - \frac{M_H^2}{4M_W^2}\left[\frac{s}{s-M_H^2} + \frac{t}{t-M_H^2}\right] + ..\right\}.$$

Comment on the high energy behaviour of $T^1_{W^+W^-\to W^+W^-} + T^2_{W^+W^-\to W^+W^-}$.

v) Now perform a partial wave expansion by letting

$$T(s,t) = 16\pi\sum_J (2J+1)a_J(s)P_J(\cos\theta)$$

where P_J are the Legendre polynomials ($P_0(x) = 1$,$P_1(x) = x$,$P_2 = (3x^2-1)/2$,..). This simply decomposes the amplitude into contributions coming from various total angular momenta J. Use the orthogonality condition

$$\int_{-1}^{1} dx P_J(x)P_K(x) = \delta_{JK}\frac{2}{2J+1}$$

and the high energy approximation to show that the total cross section is given by

$$\sigma = \frac{16\pi}{s} \sum_J (2J+1)|a_J(s)|^2.$$

Then make use of the optical theorem $\sigma = \frac{\mathbf{Im} M(s, \cos\theta=1)}{s}$ to show that $|a_J|^2 = \mathbf{Im} a_J$. Show that then $|a_J| \leq 1$.

vi) Finally use the orthogonality relation to derive that

$$a_0(s) = -\frac{G_F M_H^2}{4\pi\sqrt{2}}$$

for $s \gg M_H^2$.

vii) Use your results of v) and vi) to derive an upper bound for M_H.