Particle Physics Phenomenology II

FS 11, Series 7

Due date: 11.04.2011, 1 pm

Exercise 1 K^0 system without CP violation

The K^0 and $\bar{K}^0$ are eigenstates of strangeness. They are transformed into each other by CP transformation and are therefore not eigenstates of CP.

$$K^0 \overset{CP}{\longleftrightarrow} \bar{K}^0$$

Show that

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \tag{1}$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \tag{2}$$

are eigenstates of CP. We consider the K^0 system without CP violation. These states are decay eigenstates. The K_1^0 and K_2^0 lifetimes are T_1 and T_2. The time evolution of a state with energy E and lifetime T is given by

$$|\phi\rangle(t) = |\phi\rangle(0)\exp(-iEt - \frac{t}{2T})$$

and satisfies the Hamiltonian $\mathrm{H} = -\frac{i}{2}\Gamma$. Calculate the intensity of K^0 and $\bar{K}^0$ (i.e. $|\langle K^0|K^0(t)\rangle|^2$ and $|\langle \bar{K^0}|K^0(t)\rangle|^2$) in a beam which consists of K^0 only at $t = 0$.

Exercise 2 K^0 system with CP violation

If we consider the K^0 system with CP violation, the linear combinations K_1^0 and K_2^0 are not mass eigenstates anymore, but

$$|K_S^0\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_1^0\rangle + \epsilon|K_2^0\rangle\right) \tag{3}$$

$$|K_L^0\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_2^0\rangle + \epsilon|K_1^0\rangle\right) \tag{4}$$

are.

Consider the reactions

$$K_L^0 \rightarrow e^+\nu_e\pi^-$$
$$K_L^0 \rightarrow e^-\bar{\nu}_e\pi^+$$

with rates R_+ and R_-. Calculate the ratio

$$\delta = \frac{R_+ - R_-}{R_+ + R_-}.$$

Mind that the decays

$$\begin{aligned} A_+ \quad &: \qquad K^0 \to e^+ \nu_e \pi^- \\ A_- \quad &: \qquad \bar{K}^0 \to e^- \bar{\nu}_e \pi^+ \end{aligned}$$

are possible, with $|A_+| = |A_-|$ for the corresponding amplitudes whereas the decays

$$\begin{aligned} K^0 &\to e^- \bar{\nu}_e \pi^+ \\ \bar{K}^0 &\to e^+ \nu_e \pi^- \end{aligned}$$

are forbidden.