Particle Physics Phenomenology II

FS 11, Series 8

Due date: 18.04.2011, 1 pm

Exercise 1 This exercise concerns the theory of Neutrino oscillations in the vacuum.

i) For three generations of neutrinos, show that the transition rate from flavor α to β is given by

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re} J^{ij}_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right) - 2\sum_{i<j} \mathrm{Im} J^{ij}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$

where $J^{ij}_{\alpha\beta} = U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}$ with U being a unitary transformation matrix between the flavor and mass basis, $\Delta m^2_{ij} = m^2_i - m^2_k$,$L \simeq t$ the flight length and E the so called "spectral dependence".
Hint: Show first that

$$P_{\alpha\beta} = A^*_{\alpha\beta} A_{\alpha\beta} = \sum_{k,j=1}^{3} J^{kj}_{\alpha\beta} e^{-i(E_k - E_j)t}.$$

where $A_{\alpha\beta} = \langle \nu_\beta | \nu_\alpha(t) \rangle$, and $|\nu_\alpha(t)\rangle = \sum_{i=1}^3 U^*_{\alpha i} |\nu_i(t)\rangle$. Then use that near the massless limit $E_k \simeq E + \frac{m_k^2}{2E}$ holds.

ii) Seperate $P_{\alpha\beta}$ into a CP violating and conserving part: $P_{\alpha\beta} = P^{CPeven}_{\alpha\beta} + P^{CPodd}_{\alpha\beta}$. And show that the CP violating term is given by

$$P^{CPodd}_{\alpha\beta} = -2\sum_{i<j} \mathrm{Im} J^{ij}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right).$$

iii) Prove that the following conditions have to be satisfied for CP violation to exist, i.e. for $P^{CPodd}_{\alpha\beta} \neq 0$:

$$\alpha \neq \beta, \quad \delta \neq 0, \pi, \quad \theta_{ij} \neq 0, \quad \delta m_{ij} \neq 0, \infty.$$