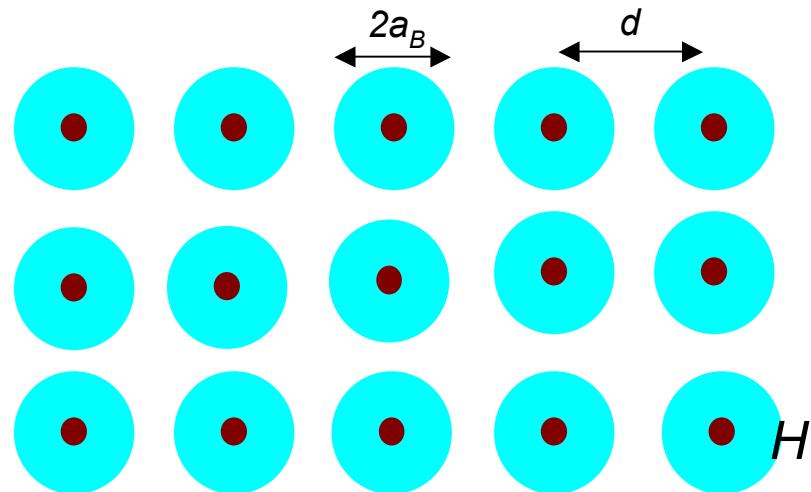


# Mott insulators

Atomic limit - view electrons in real space

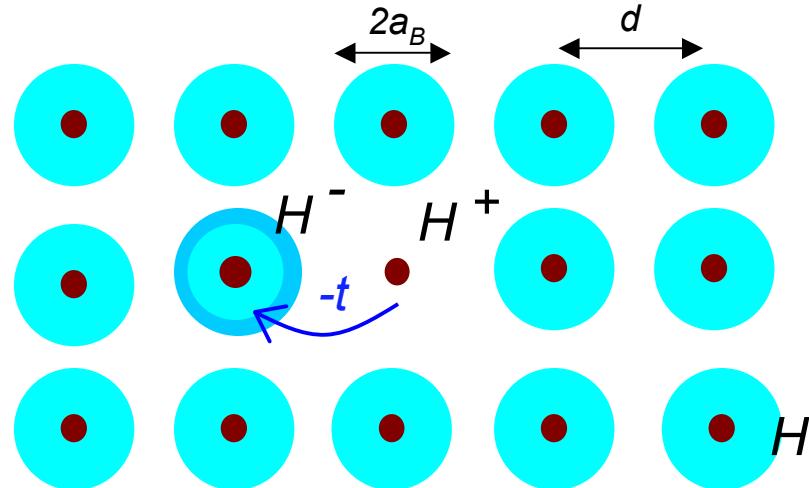
*Virtual system:* lattice of H-Atoms:  $a_B \ll d$



# Mott insulators

## Atomic limit - view electrons in real space

*Virtual system:* lattice of H-Atoms:  $a_B \ll d$



- hopping - electron transfer  
 $-t|\{H^+\}_i, \{H^-\}_j\rangle\langle\{H\}_i\{H\}_j| + h.c.$
- ionization energy  
$$U = E(H^+) + E(H^-) - 2E(H)$$

delocalization

$$-2tz$$

kinetic energy

*metal*

localization

$$U$$

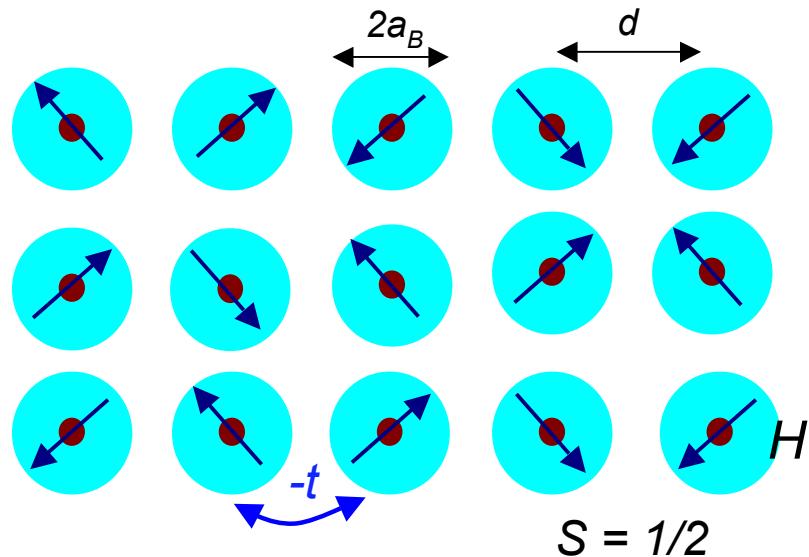
charge excitation energy

*Mott insulator*

# Mott insulators

## Atomic limit - view electrons in real space

*Virtual system:* lattice of H-Atoms:  $a_B \ll d$



### Mott isolator

low-energy physics

no charge fluctuation  
only spin fluctuation

- hopping - electron transfer  
 $-t|\{H^+\}_i, \{H^-\}_j\rangle\langle\{H\}_i\{H\}_j| + h.c.$
- ionization energy  
$$U = E(H^+) + E(H^-) - 2E(H)$$

effective low-energy model

$$H_{\text{Heisenberg}} = J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$$

# Mott insulators

# Metal-insulator transition from the insulating side

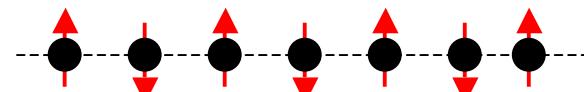
## Hubbard-model:

$$H = -t \sum_{\langle i,j \rangle, s} \left\{ c_{is}^\dagger c_{js} + c_{js}^\dagger c_{is} \right\} + U \sum_i n_{i\uparrow} n_{i\downarrow} = \sum_{\vec{k}, s} \epsilon_{\vec{k}} c_{\vec{k}s}^\dagger c_{\vec{k}s} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

n.n. hopping
onsite repulsion

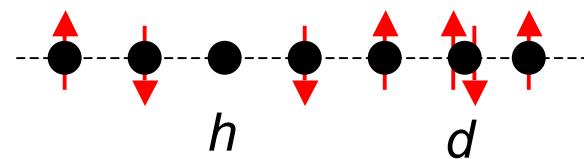
density:  $n=1$

„ground state“

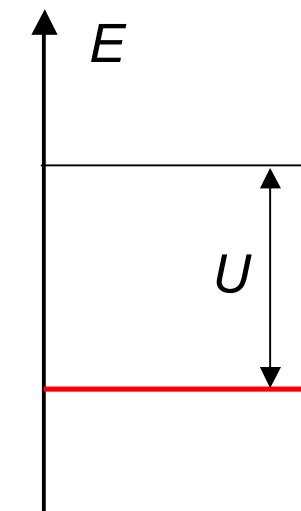


$$t = 0$$

## charge excitation



$$E = U$$



# Mott insulators

# Metal-insulator transition from the insulating side

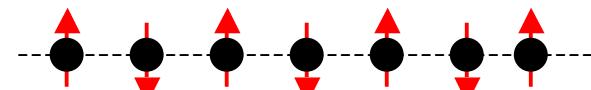
## Hubbard-model:

$$H = -t \sum_{\langle i,j \rangle, s} \left\{ c_{is}^\dagger c_{js} + c_{js}^\dagger c_{is} \right\} + U \sum_i n_{i\uparrow} n_{i\downarrow} = \sum_{\vec{k}, s} \epsilon_{\vec{k}} c_{\vec{k}s}^\dagger c_{\vec{k}s} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

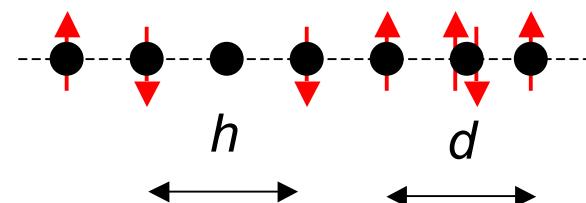
n.n. hopping
onsite repulsion

density:  $n=1$

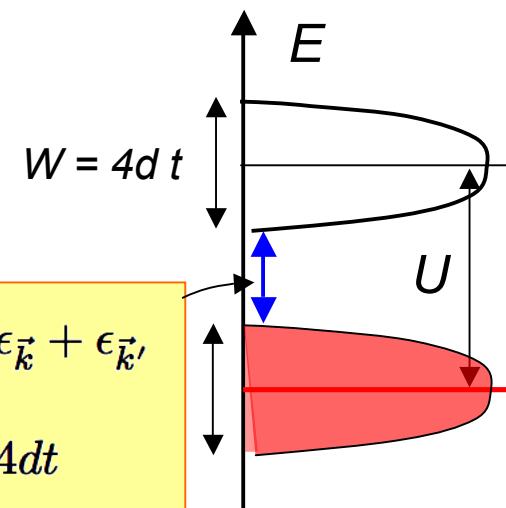
„ground state“



## charge excitation



$$E_{\vec{k}, \vec{k}'} = U + \epsilon_{\vec{k}} + \epsilon_{\vec{k}'} \\ \geq U - 4dt$$



metal-insulator transition:  $U_c = 4dt$

# Mott insulators

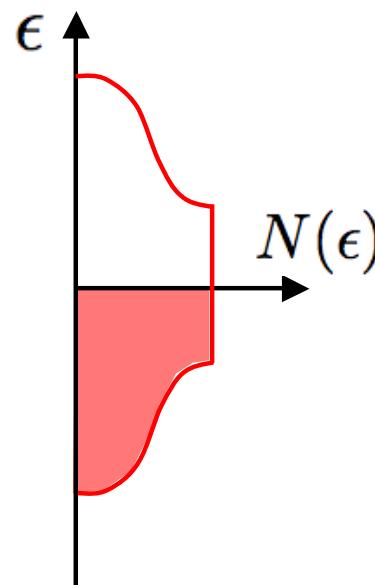
## Metal side

$$H = -t \sum_{\langle i,j \rangle, s} \left\{ c_{is}^\dagger c_{js} + c_{js}^\dagger c_{is} \right\} + U \sum_i n_{i\uparrow} n_{i\downarrow} = \sum_{\vec{k}, s} \epsilon_{\vec{k}} c_{\vec{k}s}^\dagger c_{\vec{k}s} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

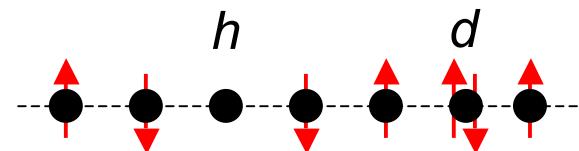
$$\epsilon_{\vec{k}} = -2t(\cos k_x a + \cos k_y a + \cos k_z a)$$

$$U = 0$$

tight-binding model



half-filled  
conduction  
band



empty sites  
doubly occupied sites  
singly occupied sites

density  
 $h = 1/4$   
 $d = 1/4$   
 $s = 1/2$

$U > 0$   $\rightarrow$   $\begin{cases} h = d \rightarrow 0 \\ s \rightarrow 1 \end{cases}$   
reducing mobility

# Mott insulators

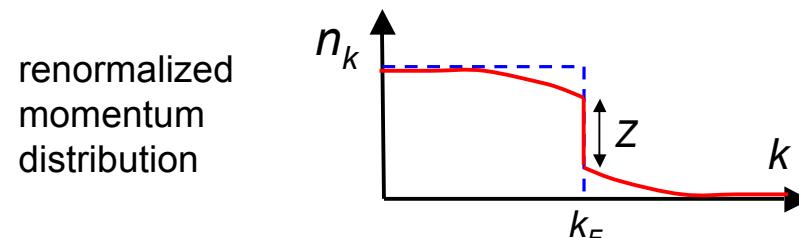
## Metal-insulator transition from the metallic side

Gutzwiller approximation

$$\mathcal{H}_{eff} = g_t \sum_{\vec{k},s} \epsilon_{\vec{k}} c_{\vec{k}s}^\dagger c_{\vec{k}s}$$

$$g_t = 1 - \left( \frac{U}{U_c} \right)^2$$

### Fermi liquid property



quasiparticle weight:  $Z = g_t$

at MIT  
 $U \rightarrow U_c$

$$\begin{cases} \frac{m^*}{m} \rightarrow \infty \\ Z \rightarrow 0 \end{cases}$$

