Solid State Theory
Exercise 6

FS 2011
Prof. M. Sigrist

Exercise 6.1 Lindhard function

In the lecture it was shown how to derive the dynamical linear response function $\chi_0(\boldsymbol{q},\omega)$ which is also known as the Lindhard function:

$$\chi_0(\boldsymbol{q},\omega) = \frac{1}{\Omega}\sum_{\boldsymbol{k}} \frac{n_F(\epsilon_{\boldsymbol{k}+\boldsymbol{q}}) - n_F(\epsilon_{\boldsymbol{k}})}{\epsilon_{\boldsymbol{k}+\boldsymbol{q}} - \epsilon_{\boldsymbol{k}} - \hbar\omega - i\hbar\eta}. \tag{1}$$

Calculate the static Lindhard function $\chi_0(\boldsymbol{q})$ of free electrons for the 1 and 3 dimensional case at $T = 0$.
Hint: We are only interested in the real part of $\chi_0(\boldsymbol{q},\omega)$. Therefore, use the equation $\lim_{\eta\to 0}(z - i\eta)^{-1} = \mathcal{P}(1/z) + i\pi\delta(z)$. Furthermore, in 3 dimensions we can choose $\boldsymbol{q} = q\boldsymbol{e}_z$ to point in the z-direction due to the isotropy of a system of free electrons. Then change to cylindrical coordinates in order to calculate the integral.

Exercise 6.2 Zero-sound excitations

The dispersion relation of the plasmon excitation is finite for all $\boldsymbol{q}$'s. This appearance of a finite excitation energy is a consequence of the long range interaction of the Coulomb potential $V_{\text{Coulomb}}(\boldsymbol{r}) = e^2/|\boldsymbol{r}|$. A system consisting of fermions with a solely local potential

$$V_{\text{local}}(\boldsymbol{r}) = U \cdot \delta(\boldsymbol{r}) \tag{2}$$

shows a different behaviour at $\boldsymbol{q} = 0$. In this exercise we basically follow the sections (3.2.1) and (3.2.2) of the lecture notes.

a) As a warm-up, derive the relation between the particle distribution $\delta n(\boldsymbol{r},t)$ and its induced potential $V_{\text{ind}}(\boldsymbol{r},t)$ in the $(\boldsymbol{k},\omega)$-space.

b) Find the imaginary part of the response function $\chi(\boldsymbol{q},\omega)$ for small $\boldsymbol{q}$'s. What is the dispersion relation in the lowest order in $\boldsymbol{q}$?

c) The upper boundary line of the particle-hole continuum is given by

$$\omega_{q,\max} = \frac{\hbar}{2m}\left(q^2 + 2k_F q\right) = \frac{\hbar q^2}{2m} + v_F q, \tag{3}$$

where v_F is the Fermi velocity and $q = |\boldsymbol{q}|$. What is the condition on U for stable plasmon excitations (quasi-particles)?

Office hour:
Monday, April 4th, 2011 - 9:00 to 11:00 am
HIT K 31.3
Tama Ma