QUANTUM THEORY
&
REALISM

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"If someone tells you they
understand quantum mechanic
then all you've learned is
that you've met a liar."

(R.P. Feynman)

"Anyone who is not shocked
by quantum theory has not
understood it."

(N. Bohr)

"We have to ask what it
means!"

(K.G. Wilson)

Contents

Credits

Coleman, Fierz, Hepp; Dürr; Gell-Mann & Hartle; Goldstein; Griffiths; Omnès; Pickl, Schilling, ...

1. Introduction

Planck, 1900:

$$\rho(\nu,T) = \frac{8\pi}{c^3}\,\nu^2\,\frac{h\nu}{e^{h\nu/kT}-1}$$

h: Planck's constant

k: Boltzmann's constant

c: speed of light

+

G_N: Newton's constant of universal gravitation

↳ $$l_P = \sqrt{\frac{G_N\,h}{c^3}}$$

(Planck length)

4 fundamental dim.-full constants of Nature, h, k, c^{-1}, l_P

$\leftrightarrow$ 4 revolutions in Physics of 20^{th} Century.

h, k	NR QT; atomism, stat. mech.	RQFT	"M-theory"?
c^{-1}	SRT		
l_P	GR		

h & c^{-1} : quantum symmetries

c^{-1} & l_P : dyn. Lorentzian geom.,

h & c^{-1} & l_P : "quantum geom." ?

Newton vs. Leibniz ... Grothend

<u>Task</u>: Complete this picture, & analyze limiting regimes of new theories:

"$\hbar \to 0$": class. (mean-field) lim

"$k \to 0$": lim of "cont. mechanics"

"$c^{-1} \to 0$": NR limit

<u>Program</u>. In which way do new theories of 20^{th} Cent. differ from precursor, "class." theories? Have I understood the new ths. I had to teach to my students?

Today: $\hbar$ & k

Atomistic QM

Next time: "Sketch of a Program"
(E.G. – A.G.)
"As Riemann pointed out, I believe, the math. continuum is a convenient fiction for dealing with phys. phenomena
Emergence of phys. space & causal structure in general (many-obs.) quantum th.

2. What is a physical system?

Realistic vs. "Idealistic" Theories

Phys. system, S, specified in terms of observable physical quantities rep. as lin. operators

$\rightarrow$ generate *alg. $\mathcal{A}_S \subseteq \mathcal{B}_S$;

$\mathcal{B}_S$: algebra of "possible events", (given some exp. equipment).

Fundamental data:

(I) $\mathcal{A}_S \subseteq \mathcal{B}_S$: a C^*-alg. (dep. on $\mathcal{O}$)

(II) $\mathcal{S}_S$: "states" on $\mathcal{B}_S$ (standard)

(III) G_S: "symmetries" of S, incl. time evolution.

Symm. trsfs. from G_S act as *automorphisms on $\mathcal{B}_S$; ex: time evolution: $(t,s) \mapsto \alpha_{t,s} \in Aut(\mathcal{B}_S)$

(IV) Subsyst./composition:

$$S \subset S' \Rightarrow \mathcal{B}_S \subset \mathcal{B}_{S'}$$

$$(S_1, S_2) \mapsto S_1 \vee S_2 \text{, with } \mathcal{B}_{S_1 \vee S_2} = \mathcal{B}_{S_1} \otimes \mathcal{B}_{S_2}$$

If $S_1 \simeq S_2 \simeq S$ specify imbedding

$$\mathcal{S}_{S \vee S} \hookrightarrow \mathcal{S}_S \otimes \mathcal{S}_S \text{ : statistics}$$

Choice of (I), (II), (III) depends on equipment available to observe Nature, O (observer).

New theories arise by "deformations" of (I), (III), (IV); (Flato, Faddeev)

(I)
- cont. ths. of matter $\xrightarrow{k}$ atomism
- class. mechanics $\xrightarrow{h}$ QM

(III) Galilei symm. $\xrightarrow{c^{-1}}$ Poincaré $\xrightarrow{R^{-1}}$ de Sitter

(IV)
- permutation stat. $\rightarrow$ braid stat
- group symm. $\rightarrow$ quantum group.

$\updownarrow$

th. of braided $\otimes$ categories, duality (Tannaka-Krein th.)

<u>Ex.</u> Vlasov th. $\xrightarrow{k}$ Newtonian mech

$\downarrow h$

wave mechanics

realistic ("class.") theories R

"idealistic" (quantum) ths. Q

(R) Realistic theories

- $\mathcal{B}_S$ abelian $\underset{\text{Gel'fand}}{\Longrightarrow}$ $\mathcal{B}_S \simeq C_0(M_S)$

  $M_S = \operatorname{spec} \mathcal{B}_S$ (e.g., $M_S = \Gamma$)

- $\mathcal{S}_S = \{$prob. measures on $M_S\}$

  Pure States $= \{\delta$-fns. on $M_S\}$

  $\simeq \{$chars. of $\mathcal{B}_S\}$

  $\updownarrow$

  no superposition principle;

  no entanglement betw. S_1 & S_2 in $S_1 \vee S_2$.

- *automorphisms of $\mathcal{B}_S$

  $\overset{1-1}{\longleftrightarrow}$ homeomorphisms of M_S

<u>Problem</u>. When does TM_S exist (is M_S a diff. (sympl. ...) mf.)? If it does then time evol. $\{\alpha_{t,s}\}$ generated by VF, X_t, on M_S:

$$\dot{\xi}_t = X_t(\xi_t), \quad \xi_t \in M_S.$$

$\to$ Realistic & det. descr. of S!

$$P_i := \alpha_{t_i,t_0}(\chi_{\Omega_i}) = \chi_{\Omega_i} \circ \phi_{t_i,t_0} \doteq \chi_{\phi^{-1}_{t_i,t_0}(\Omega_i)}$$

Then, for $\xi_0 \in M_S$,

$$\delta_{\xi_0}\left(\prod_{i=1}^{n} P_i\right) = 0 \text{ or } 1 \,!$$

<u>"Effective" dynamics</u>:

$$T_{t,s} : \mathcal{S}_S \to \mathcal{S}_S, \quad \text{w. } T_{t,s} \circ T_{s,u} = T_{t,u}$$

$\to$ Stoch. processes on M_S!

(Q) Quantum theories

$\mathcal{A}_S$, hence $\mathcal{B}_S$, non-abelian

Example: $\mathcal{B}_S$ type-I C^*-alg.
(e.g. group alg. of compact Lie group – qm spins – or Weyl for S w. finite nb. of degs. of freedom)

$\mathcal{Z}_S :=$ centre of $\mathcal{B}_S$: abelian → R

- $$\mathcal{B}_S \simeq \int^{\oplus}_{\mathrm{spec}\, \mathcal{Z}_S} B(\mathcal{H}_\xi)\,, \quad \xi \in \mathrm{spec}\, \mathcal{Z}_S\,,$$

$\mathcal{H}_\xi$: Hilbert space

$\mathcal{S}_S = \{\text{density matrices on } \mathcal{H}.\} \otimes \{\text{prob. meas. on spec } Z_S\}$

Pure states

$= \{\text{unit rays in } \mathcal{H}_\xi \mid \xi \in \text{spec } Z_S\}$

- Superposition princ. within every $\mathcal{H}_\xi$
- Entanglement betw. S_1 & S_2 in $S_1 \vee S_2$

$\mathcal{A}_S$ non-commutative

↙ uncertainty rels. (Heisenberg)

↘ decoh. (S "cat") entanglement (Schrödinger)

↓

Bell <, CHSH <, (Kochen-)Specker

↑ (Bell <, CHSH <) — Tsirelson ← Grothendieck

Theory intr. probabilistic

→ sep. lect.

(no realistic int. of QM → 3; non-extendability - Colbeck & Renner)

- Dynamics $\{\alpha_{t,s}\}$ → unitary propagators on $\mathcal{H}$. × flow on spec Z_S.

Eff. dynamics: "Lindbladians"

Preparation: S can be "prepared" in pure states $[\psi]$, $\psi \in \mathcal{H}_\xi$, without further info. on initial conditions. In exp., observable $a = a^* \in \mathcal{A}_S$ can be measured with $[a, P_\psi] \neq 0$.

"Attracting families of states"

→ separate lecture!

3. Why must a "realistic" interpretation of QM fail? The no-signaling lemma

Realistic (det.) int. of QM: ...

QM does sometimes predict facts: ... (Theory of preparation of (sub-) systems)

Yet, there remains an irred. element of chance, whenever A_S is non-abelian.

Example (no-signaling lemma-F-P-S) 2 electrons prep. in spin singlet state:

What would a realistic int. of QM be?

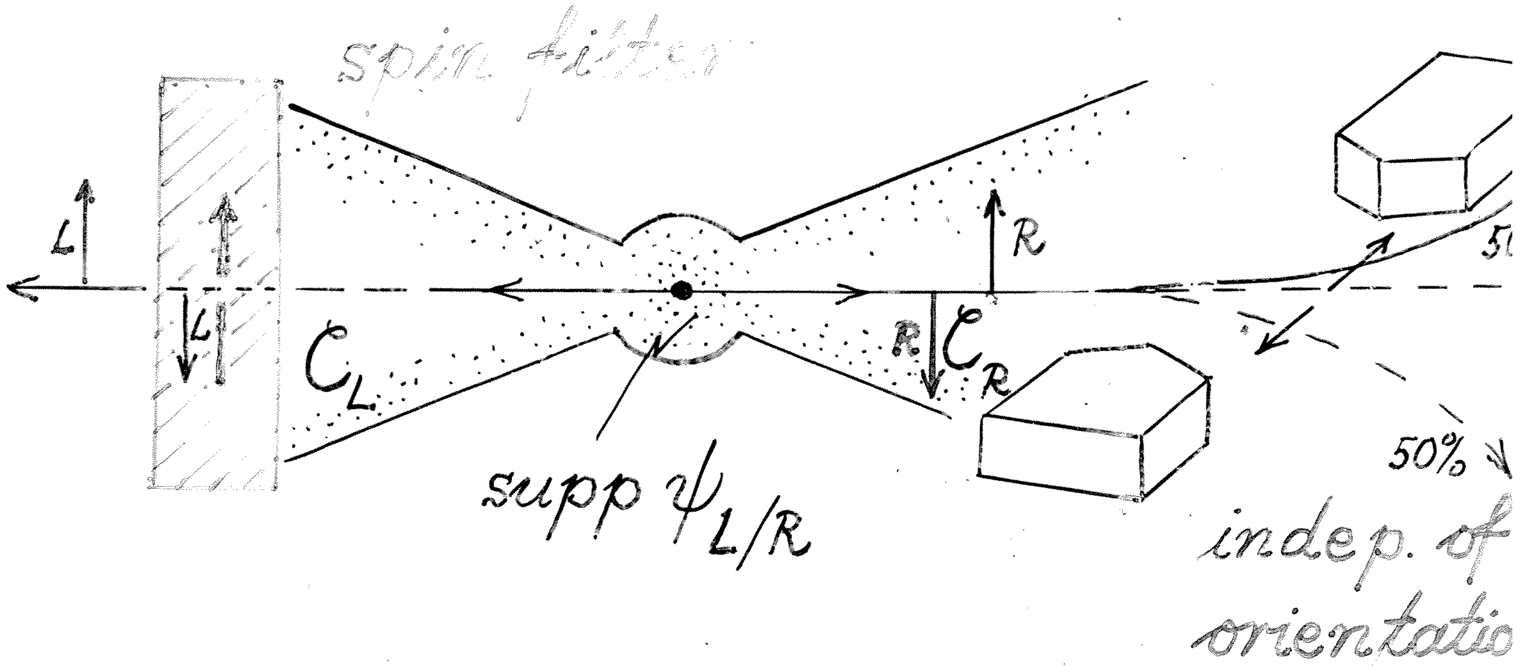
"Wave function of universe" determines "everything" observable for humans – after "Penrose correction" (collapse of wave fn. due to gravity).

Now, QM does sometimes predict facts/or events with certainty:

- values of superselected phys. quantities*;
- convergence of states of subsystems to "attractors" ("relaxation to ground-states", return to equilibrium, ...) → theory of preparation of quantum systems

Yet: Realistic (deterministic) int. of a theory with A_S non-abelian is impossible! There remains an irreducible element of chance.

$$\Psi^{(2)}_{\cdot} := (\psi_L \otimes \psi_R + \psi_R \otimes \psi_L) \otimes (|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle)$$



$_L\!\uparrow$ transm., $\downarrow^L$ absorbed

<u>Experiment</u>: If $_L\!\uparrow$ observed

then $_R\!\downarrow$ predicted; (1)

if $\downarrow^L$ obs. <u>then $\uparrow^R$</u>, predicted

$\psi_{L/R}$ propagates into $C_{L/R}$, except for tiny tails.

Initial state of composed syst.:

$$\Phi_0 = \sum_\alpha \Psi^{(2)}_\alpha \otimes \chi_{filter,\alpha}$$

Dynamics: $H = H_0 + H_I$

H_0 : dyn. of uncoupled syst.

H_I : int. electrons-filter
localized around filter.

$$\Phi_t := e^{-itH}\,\Phi_0$$

<u>Lemma</u> ("no signaling")

Under "reasonable hyp." on H_I,

$$\langle \Phi_t, \vec{S}_{\mathcal{C}_R} \Phi_t \rangle \simeq 0, \qquad (2)$$

for all t.

Consequ. of Cook arg. & "cluster props." – Suppose that (R)

$$\langle \Phi_t, \vec{S}_{\mathcal{C}_L} \Phi_t \rangle = (\hbar/2)\,\vec{e}_3 \qquad (L\uparrow)^*$$

(2) & $(L\uparrow)^*$, or $(\downarrow L)$, contradict (1)!

⇒ $(L\uparrow)^*$, i.e. (R), impossible!

⇒ Frequ. of transm. of left el. through filter $\simeq 1/2$, ...

Thus, Φ_t does not describe what happens in a det. way, but only what may happen. Einstein causality not invoked

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→ J.F., P.P., C.S.

4. Some fundamental notions and questions about QM

System, S, (e.g., e^-, atom, ...) to be explored w. exp. equipment, E, (e.g., lasers, magnets, detectors, filters, ...).

$S \vee E$ described by ($\nearrow$ 2, (I)):

(I) C^*-alg. $\mathcal{B} \equiv \mathcal{B}_{S \vee E}$, "states" $\mathcal{S}\ (\ni \omega, \rho, \cdots)$ on $\mathcal{B}$

(II) Time evolution, $\{\alpha_{t,s}\}_{t,s \in \mathbb{R}}$, in Heisenberg picture (*automorphisms of $\mathcal{B}$)

(III) $\mathcal{B} \supset \mathcal{A}_S$: v. Neumann alg.

generated by operators repr. <u>phys.</u> <u>quantities</u> referring to S at some time t_0, (comp. of position, momentum, spin of particle belonging to S ..., at time t_0).

For obs. $a = a^* \in \mathcal{A}_S$,

$$a_t := \alpha_{t,t_0}(a) \in \mathcal{B}$$

denotes obs. corresp. to a at time t.

$P_{a_t}(I)$: spect. proj. of a_t corresp. to $I \subset \mathbb{R}$.

$$P_{a_t}(I)^{\perp} := P_{a_t}(I^c) = \mathbb{1} - P_{a_t}(I)$$

Questions to be raised:

(1) How can one prepare spec. initial states, ω, of $S \vee E$ at time t_0 when exps. start? (Th. of prep. $\leftrightarrow$ attracting fams of states $\leftrightarrow$ relaxation to metastable states & g.s.: sep. lect.)

(2) Exps. on S, using E, done to produce "obj. events/facts", e.g., measured values of obs. $a_t = a_t^*$, $a \in \mathcal{A}_S$. — What are "possible events" in QM?

"Possible event" at time t $\leftrightarrow$ spect. proj. $P \equiv P_{a_t}(I)$, $a \in \mathcal{A}_S$.

"<u>Ontology</u>" underlying QM:
Time-ordered sequs. of "events"
= "<u>histories</u>" triggered by E.
E, $\{\alpha_{t,s}\}$ chosen such that
class of events $\{P_{a_t}(I)\}_{I \subset \mathbb{R}} \leftrightarrow$
"super sel. sectors" of E
(~"pointer positions") identifiable in non-demolition measurements of E.

$\Rightarrow$ No ∞ sequ. $E \leftarrow E' \leftarrow E'' \leftarrow \cdots$
needed to trigger events!

(3) Rôle of E in rendering possible events $P \equiv P_{a_t}(I)$ & $P^{\perp}$ complementary, given sub-

sequent events ($\nearrow$ Weyl) $\rightarrow$ "Evidence for events to have happened", given the future.

(4) Can QM predict frequencies (emp. probs.) of possible "histories", and how?

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5. Calculus of frequencies

(Lüders, Schwinger, Wigner)

$\{P_n, P_{n-1}, \cdots, P_1\}$: time-ordered history of possible events,

$$P_i := P_{a^i_{t_i}}(I_i),$$

$a^i = (a^i)^* \in \mathcal{A}_S$, $t_0 < t_1 < t_2 < \cdots < t_n$, $I_i \subset \mathbb{R}$, given E, $\{\alpha_{t,s}\}$.

QM predicts "frequency"/emp. prob. of $\{P_n, P_{n-1}, \dots, P_1\}$, given an initial "state" ω of $S \vee E$ at time t_0.

"Master formula"

$$\mathcal{F}_\omega\{P_n, \dots, P_1\} := \omega\left(P_1 P_2 \cdots P_{n-1} P_n P_{n-1} \cdots P_2 P_1\right) \qquad (3)$$

Properties of $\mathcal{F}_\omega$

(i) $\mathcal{F}_\omega\{P_n, \dots, P_1\} \geq 0$,

(ii) Set $P_j^+ := P_j$, $P_j^- := P_j^\perp = \mathbb{1} - P_j$.

$$\sum_{\substack{\varepsilon_j = \pm \\ j = 1, \dots, n}} \mathcal{F}_\omega\{P_n^{\varepsilon_n}, \dots, P_1^{\varepsilon_1}\} = 1$$

$$\Longrightarrow \quad 0 \leq \mathcal{F}_\omega\{P_n, \dots, P_1\} \leq 1 \qquad (4)$$

Remark. For realistic syst. S, F_ω obeys a 0-1 Law if ω pure

(iii) "Symmetry between prediction & retrodiction" (A-B-L).

(iv) Complementarity of "events"

$$F_\omega\{P_n, \dots, P_j, \dots, P_1\} + F_\omega\{P_n, \dots, P_j^\perp, \dots, P_1\}$$

$$\underset{i.g.}{\neq} F_\omega\{P_n, \dots, P_{j+1}, P_{j-1}, \dots, P_1\}, \quad (5)$$

unless $j = n$, because, i.g.,

$$\omega(P_1 \cdots P_j \cdots P_n \cdots P_j^\perp \cdots P_1) \underset{i.g.}{\neq} 0 \quad (6)$$

"interference"

→ No meaningful ("non-contextual") notion of "conditional probability" of P_j, given future.

A possible event, P_j, can become a fact iff it does not interfer with its compl., $P_j^\perp$, given future "Evidence" for P_j or $P_j^\perp$ to happen:

$$\mathcal{E}_\omega(P_j \,|\, \{P_j\}^c) :=$$

$$1 - \max_{\varepsilon_k, k \geq j} \left| \omega(P_1 \cdots P_j \cdots P_n^{\varepsilon_n} \cdots P_j^\perp \cdots P_1) \right|$$

History $\{P_n, \dots, P_1\}$ is δ-consistent w.r. to ω iff

$$\mathcal{E}_\omega(P_j \,|\, \{P_j\}^c) \geq 1 - \delta, \ \forall j. \quad (7)$$

Only δ-consistent histories, w. $\delta \ll 1/n$, represent a useful concept, admitting a "class." interpretation, FAPP.

Example $(n=2)$: 2-slit exp. with screen & laser lamp. How do δ-consistent histories arise $(\delta \ll 1/n)$?

6. Dephasing & decoherence

Consider history $\{P_n, \dots, P_1\}$, w.

$$P_i := P_{a^i_{t_i}}(I_i), \; a^i = (a^i)^* \in \mathcal{A}_S,$$

$$I_i \subset \mathbb{R}, \; t_n > t_{n-1} > \dots > t_1 > t_0.$$

At some time $t_< \geq t_{j-1}$, measurement of a^j starts (choice of E & of interactions $S \leftrightarrow E$ after time $t_<$), with possible outcome $P_j = P_{a^j_{t_j}}(I_j)$ at time $t_j > t_<$.

Under what conditions can P_j be considered a fact, given that at times $t_n > \cdots > t_{j+1} > t_j$, events $P_n, \dots, P_{j+1}$ may happen?

Events complementary to P_j:

$$P_j^{\perp} = \mathbb{1} - P_j = \sum_{\ell} P_j^{\perp,\ell} \leftarrow \text{spect. proj. of } a^j_{t_j}$$

ρ: State of $S \vee E$ at time $t_<$.

P_j a "fact" iff

$$\mathcal{F}_\rho\{P_n, \dots, P_{j+1}, P_j\} + \sum_{\ell} \mathcal{F}_\rho\{P_n, \dots, P_{j+1}, P_j^{\perp,\ell}\}$$
$$\simeq \mathcal{F}_\rho\{P_n, \dots, P_{j+1}\} \qquad (8)$$

Sufficient condition for (8):

- $\overline{\mathcal{A}}_S := \langle b := \prod_i \alpha_{t_i, t_0}(a_i) \in \mathcal{B}, \; a_i \in \mathcal{A}_S, \forall i \rangle$
- $\tau_t : \overline{\mathcal{A}}_S \longrightarrow \overline{\mathcal{A}}_S$, given by

$$\tau_t(b) := \prod_i \alpha_{t_i + t, t_0}(a_i)$$

<u>Dephasing</u> (given E):

$$\rho(\tau_t(b)) = \rho(P_j \tau_t(b) P_j) + \sum_\ell \rho(P_j^{\perp,\ell} \tau_t(b) P_j^{\perp,\ell}) + \mathcal{E}_{t,\Lambda}$$

with $\mathcal{E}_{t,\Lambda} \to 0$, as $t \to \infty$ & in TD limit, $\Lambda \nearrow \mathbb{R}^3$, of E.

For $t < \infty$, dephasing can be "undone" by operations on E! (Weyl)

Sufficient for dephasing, but stronger, is:

<u>Decoherence</u> (given E):

$$[\alpha_{t_j, t_0}(a_j), \tau_t(b)] \xrightarrow{w} 0,$$

as $t \to \infty$ & in TD lim of E;

(e.g., τ_t "asy. abelian" on $\bar{A}_S$).

Given only measurements at much later times, $t \to \infty$, $\alpha_{t_j, t_0}(a_j)$ becomes "central".

As $t \to \infty$, decoherence cannot be undone by ops. on E.

Assuming dephasing or decoherence, frequency of event P_j approaches $\rho(P_j)$, indep. of later measurements at much later times.

"Born's Rule"

"Almost classical (→ consistent) histories" are close to "class. histories", in following sense:

Given hist. $\{P_n, \cdots, P_1\}$ with

$$\|[P_i, P_j]\| < \varepsilon, \ \forall i, j;$$

then $\exists\, C_n < \infty$ & hist. $\{\tilde{P}_n, \cdots, \tilde{P}_1\}$ s.t. $\tilde{P}_i$ an orth. proj. with

$$\|\tilde{P}_i - P_i\| < C_n \varepsilon, \ \forall i, \ \&$$

$$[\tilde{P}_i, \tilde{P}_j] = 0, \ \forall i, j.$$

Models for dephasing & decoherence: Primas, Coleman–Hepp (Fierz); Zeh, ..., Dürr et al., Balian et al., J.F. et al., C.S., ... :

Quantum theory of systems with ∞ many degs. of freedom.
