Solid State Theory
Exercise 6

Exercise 6.1 Lindhard function

In the lecture it was shown how to derive the dynamical linear response function $\chi_0(\boldsymbol{q}, \omega)$ which is also known as the Lindhard function:

$$\chi_0(\boldsymbol{q}, \omega) = \frac{1}{\Omega} \sum_{\boldsymbol{k}} \frac{n_F(\epsilon_{\boldsymbol{k}+\boldsymbol{q}}) - n_F(\epsilon_{\boldsymbol{k}})}{\epsilon_{\boldsymbol{k}+\boldsymbol{q}} - \epsilon_{\boldsymbol{k}} - \hbar\omega - i\hbar\eta}. \tag{1}$$

Calculate the static Lindhard function $\chi_0(\boldsymbol{q})$ of free electrons for the 1 and 3 dimensional case at $T = 0$.
Hint: We are only interested in the real part of $\chi_0(\boldsymbol{q}, \omega)$. Therefore, use the equation $\lim_{\eta \to 0}(z - i\eta)^{-1} = \mathcal{P}(1/z) + i\pi\delta(z)$. Furthermore, in 3 dimensions we can choose $\boldsymbol{q} = q\boldsymbol{e}_z$ to point in the z-direction due to the isotropy of a system of free electrons. Then change to cylindrical coordinates in order to calculate the integral.

Exercise 6.2 Zero-sound excitations

The dispersion relation of the plasmon excitation is finite for all $\boldsymbol{q}$'s. This appearance of a finite excitation energy is a consequence of the long range interaction of the Coulomb potential $V_{\text{Coulomb}}(\boldsymbol{r}) = e^2/|\boldsymbol{r}|$. A system consisting of fermions with a solely local potential

$$V_{\text{local}}(\boldsymbol{r}) = U \cdot \delta(\boldsymbol{r}) \tag{2}$$

shows a different behaviour at $\boldsymbol{q} = 0$. In this exercise we basically follow the sections (3.2.1) and (3.2.2) of the lecture notes.

a) As a warm-up, derive the relation between the particle distribution $\delta n(\boldsymbol{r}, t)$ and its induced potential $V_{\text{ind}}(\boldsymbol{r}, t)$ in the $(\boldsymbol{k}, \omega)$-space.

b) Find the imaginary part of the response function $\chi(\boldsymbol{q}, \omega)$ for small $\boldsymbol{q}$'s. What is the dispersion relation in the lowest order in $\boldsymbol{q}$?

c) The upper boundary line of the particle-hole continuum is given by

$$\omega_{q,\max} = \frac{\hbar}{2m}\left(q^2 + 2k_F q\right) = \frac{\hbar q^2}{2m} + v_F q, \tag{3}$$

where v_F is the Fermi velocity and $q = |\boldsymbol{q}|$. What is the condition on U for stable plasmon excitations (quasi-particles)?

This collective mode has been predicted by Landau in 1957 in the framework of his theory for Fermi liquids. In 1966 zero sound was experimentally observed in He^3 by Abel, Anderson and Wheatley. References :

- L. D. Landau, JETP 32, 59 (1957), Soviet Phys. JETP 5, 101 (1957).
- W. R. Abel, A. C. Anderson, and J. C. Wheatley, Propagation of Zero Sound in Liquid He^3 at Low Temperatures, Phys. Rev. Lett. 17, 7478 (1966).
- L. P. Pitaevskii, Zero Sound in Liquid He^3, Sov. Phys. Usp. , **10, 100 (1967).**

Office hour:
Monday, April 2th, 2012 - 09:00 to 11:00 am
HIT K 12.2
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