ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

# Solid State Theory
# Exercise 12

SS 12
Prof. M. Sigrist

### Exercise 12.1 Conductivity tensor

Calculate explicitly the conductivity tensor $\sigma_{\alpha\beta}$ for a dispersion relation of the form

$$\varepsilon_{\boldsymbol{k}} = \sum_{\alpha} \frac{\hbar^2 k_\alpha^2}{2m_\alpha} \tag{1}$$

in the static limit and at $T = 0$.

### Exercise 12.2 Residual Resistivity

Recapitulate the section 6.3.1 and try to find an explanation for the data in Table 1. What is the major reason for the increase of the resisitivity?

| Impurity | Resistivity (per 1% of impurity atoms) $\rho/(10^{-8}\Omega m)$ |
|---|---|
| Be | 0.64 |
| Mg | 0.6 |
| B | 1.4 |
| Al | 1.2 |
| In | 1.2 |
| Si | 3.2 |
| Ge | 3.7 |
| Sn | 2.8 |
| As | 6.5 |
| Sb | 5.4 |

Table 1: Residual resistivity of Cu for different impurities (From Landolt-Börnstein Tables, Vol 15, Springer, 1982)

**Hint:** In analogy to doped semiconductors, assume that an impurity atom will adjust itself corresponding to its neighbourhood, i.e. it rejects all electrons from those shells which are not occupied by the surrounding atoms such that an effective charge of the nucleus remains.

**Exercise 12.3 Magnetoresistance and Hall effect**

We want to consider the electrical resistivity in the presence of a stationary magnetic field $\boldsymbol{B}$. The linearized Boltzmann equation in this case reads

$$-e\boldsymbol{v}_{\boldsymbol{k}} \cdot \boldsymbol{E}\frac{\partial f_0(\varepsilon_{\boldsymbol{k}})}{\partial \varepsilon_{\boldsymbol{k}}} - \frac{e}{\hbar}(\boldsymbol{v}_{\boldsymbol{k}} \times \boldsymbol{B})\frac{\partial g(\boldsymbol{k})}{\partial \boldsymbol{k}} = -\frac{g(\boldsymbol{k})}{\tau(\varepsilon_{\boldsymbol{k}})} \tag{2}$$

where $g(\boldsymbol{k}) = f(\boldsymbol{k}) - f_0(\boldsymbol{k})$.

a) Why is in (2) the linear order term with magnetic field coupled to the difference $g(\boldsymbol{k})$ and not to the equilibrium distribution function $f_0(\boldsymbol{k})$ (as it is in the case of electric field)?

b) Calculate for the case of $\boldsymbol{B} = (0, 0, B)$ the resistivity tensor $\hat{\rho} = \hat{\sigma}^{-1}$ and show that

   (i) the Hall resistance becomes independent of the scattering time $\tau$ and that

   (ii) the transverse magnetoresistance defined by

$$\Delta\rho_{xx}(B) = \rho_{xx}(B) - \rho_{xx}(0) \tag{3}$$

   is equal to zero.

   **Hint:** Consider independently the cases where the electric field points in the $x$, $y$ and $z$ direction, respectively, and use the ansatz

$$g(\boldsymbol{k}) = -\frac{\partial f_0}{\partial \epsilon}(ak_x + bk_y). \tag{4}$$

**Office hour:**
Monday, May 21th, 2011 - 10:00 to 12:00 am
HIT G 33.1
Jakub Imriska