Sheet 3

Due date: 16 March 2012

Exercise 1 [*Multipole expansion*]:

(i) Consider a charge configuration with Cartesian multipole moments q, $\mathbf{p}$ und Q_{ij} with respect to a coordinate system S, and Cartesian multipole moments $\hat{q}$, $\hat{\mathbf{p}}$ and $\hat{Q}_{ij}$ with respect to a coordinate system $\hat{S}$ shifted by the vector $\mathbf{R}$ relatively to S. The coordinate axis of S and $\hat{S}$ are assumed to be parallel. What is the relation between the monopole-, dipole- and quadrupole moments in the two coordinate systems?

(ii) If $q \neq 0$, can $\mathbf{R}$ be chosen such that $\hat{\mathbf{p}} = 0$? Moreover, if $q \neq 0$ and $\mathbf{p} \neq \mathbf{0}$, can $\mathbf{R}$ be chosen such that $\hat{Q}_{ij} = 0$?

Exercise 2 [*Magnetic field of an inductor*]: Consider an inductor with radius R and length L along the z-axis (the rotation axis of the inductor). Furthermore, let n be the number of windings per unit length, and I the electric current through the inductor. Compute the z-component of the magnetic field for an arbitrary point on the z-axis, and determine the magnetic field for $L \to \infty$ with n held fixed.

[**Hint**: $\int dx \, \frac{1}{\sqrt{x^2+w^2}^{\,3}} = \frac{x}{w^2 \cdot \sqrt{x^2+w^2}}$.]

Exercise 3 [*Force between a wire and a conductor loop*]: Consider an infinitely long, straight wire and a conductor loop with radius a, both lying in the x-y-plane. What is the force $\vec{F}$ on the conductor loop and the wire if b is the distance between the center of the conductor loop and the wire (with $b > a$), I_1 the electric current through the wire, and I_2 the current through the conductor loop?

[**Hint**: Use the integral of exercise 2 as well as $\int_0^{2\pi} dt \, \frac{1}{s+\cos t} = \frac{2\pi}{\sqrt{s^2-1}}$, for $s > 1$.]