Exact Entanglement Transformations

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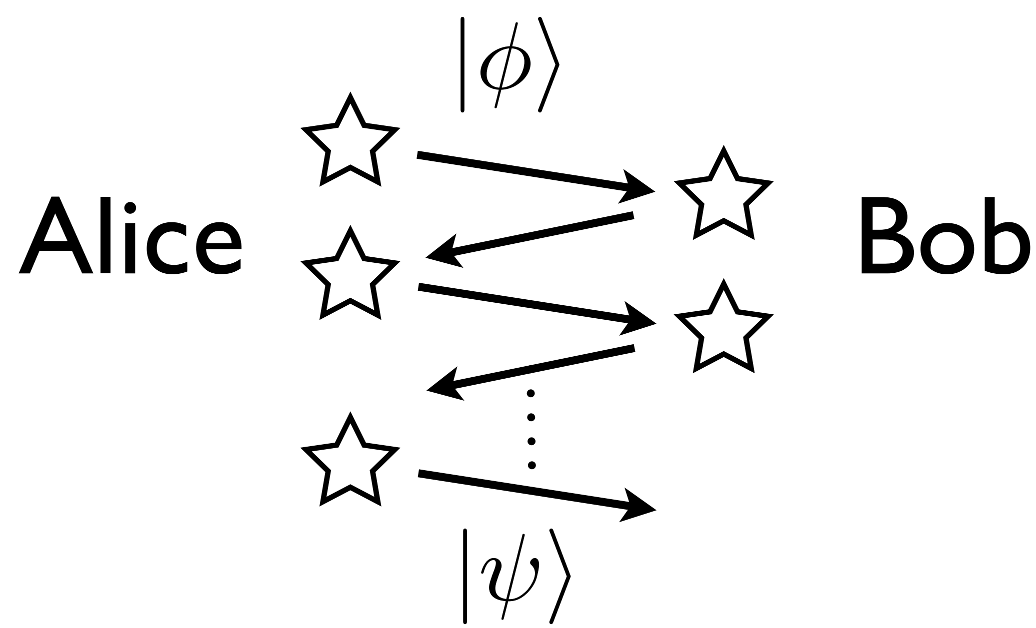
LOCC

Goal: given two states, understand which state is „more“ entangled

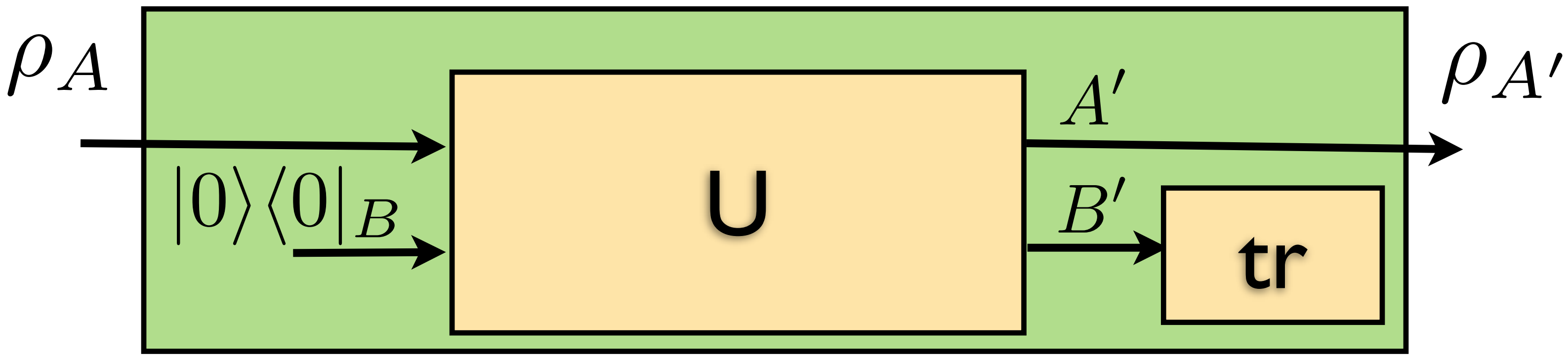
Tool: study transformation of quantum states under LOCC

local operations and classical communication

If $|\phi\rangle$ can be transformed into $|\psi\rangle$ then $|\phi\rangle$ is more entangled than $|\psi\rangle$



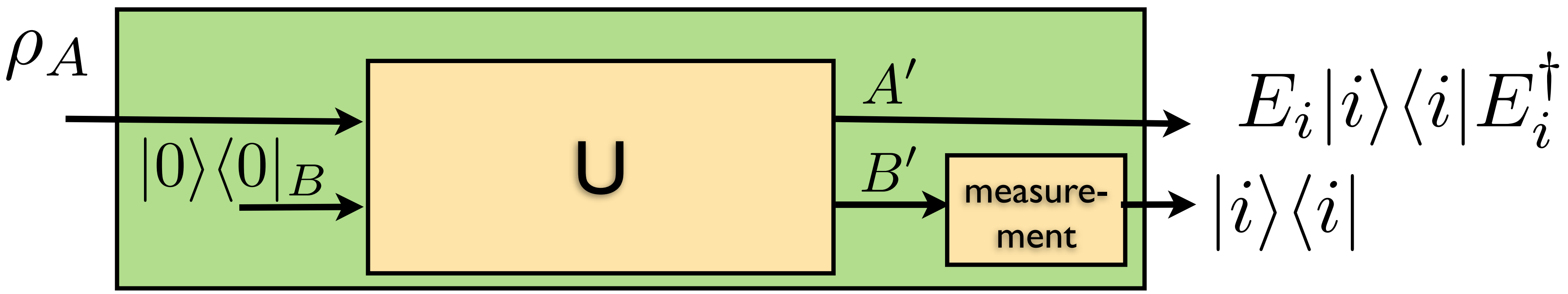
CPTP maps



$$\Lambda(\rho_A) = \mathrm{tr}_{B'} U(\rho_A \otimes |0\rangle\langle 0|_B) U^\dagger = \sum_i \langle i|_{B'} U |0\rangle_B \rho_A \langle 0|_B U^\dagger |i\rangle_{B'}$$

$$= \sum_i E_i \rho_A E_i^\dagger$$

Kraus operators:
matrices, mapping A into A'

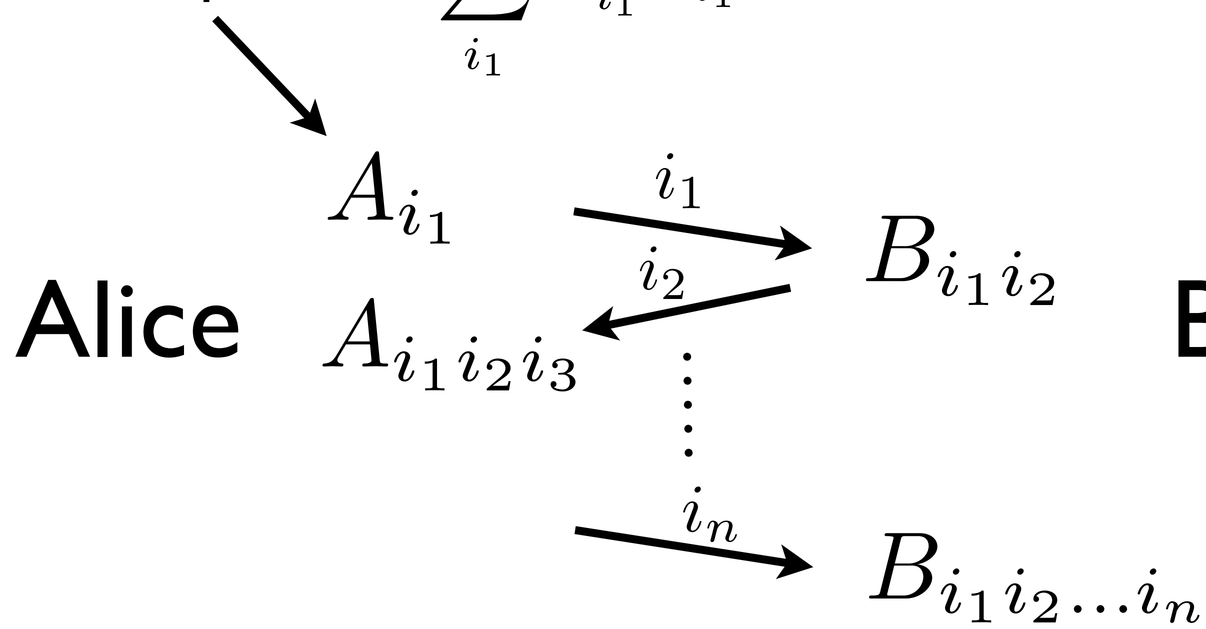


$$\{|i\rangle\langle i|\}_i$$

$$\Lambda(\rho_A) = \sum_i E_i \rho_A E_i^\dagger \otimes |i\rangle\langle i|$$

LOCC

Kraus operators $\sum_{i_1} A_{i_1}^\dagger A_{i_1} = id$



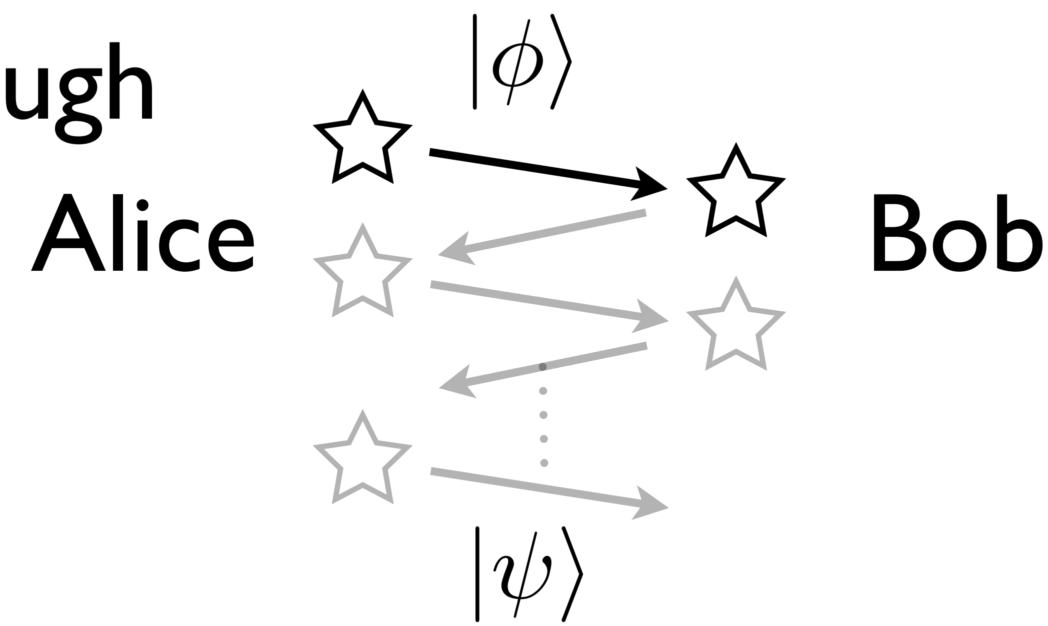
$$\Lambda(\rho) = \sum_{i_1,\ldots i_n} A_{i_1\ldots i_{n-1}} \cdots A_{i_1} \otimes B_{i_1\ldots i_n} \cdots B_{i_1 i_2} \rho A_{i_1}^\dagger \cdots A_{i_1\ldots i_{n-1}}^\dagger \otimes B_{i_1 i_2}^\dagger B_{i_1\ldots i_n}^\dagger \otimes |i_1 \ldots i_n\rangle\langle i_1 \ldots i_n|_C$$

super-complicated

Pure state transformations

Popescu et al.

Lemma: 1-round LOCC is enough



Nielsen

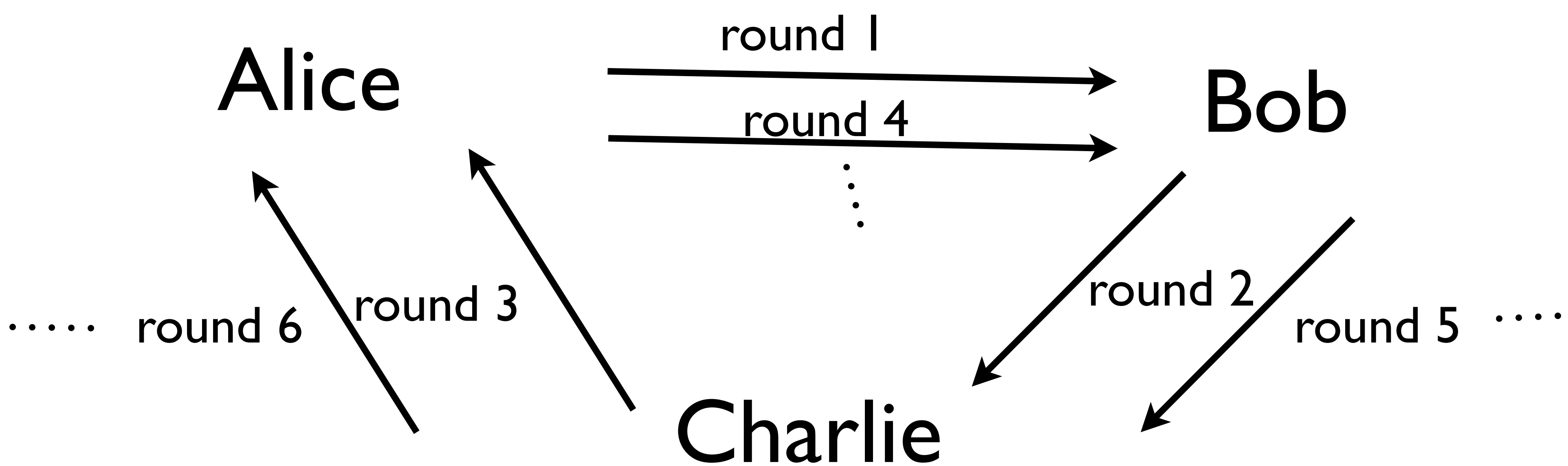
Theorem: $|\phi\rangle \xrightarrow{\text{LOCC}} |\psi\rangle$ iff $x \prec y$

x: eigenvalues of reduced state of $|\phi\rangle$

y: eigenvalues of reduced state of $|\psi\rangle$

$$\forall k : \sum_i^k x_i \leq \sum_i^k y_i$$

Multiparty LOCC



$$\Lambda(\rho) = \sum_{i_1,\ldots i_n} A_{i_1\ldots i_{n-2}} \cdots A_{i_1} \otimes B_{i_1\ldots i_{n-1}} \cdots B_{i_1 i_2} \otimes C_{i_1\ldots i_n} \cdots C_{i_1 i_2 i_3} \rho(\ldots)^\dagger \otimes |i_1 \ldots i_n\rangle\langle i_1 \ldots i_n|$$

super-duper-
complicated

SLOCC

stochastic

post-select on one set of outcomes $i_1 \dots i_n$

$$\Lambda^{post}(\rho) = A \otimes B \otimes C \rho A^\dagger \otimes B^\dagger \otimes C^\dagger$$

probability of success $tr\Lambda^{post}(\rho)$

can be very small

Def: $|\phi\rangle \xrightarrow{\text{SLOCC}} |\psi\rangle$ if $|\psi\rangle = A \otimes B \otimes C|\phi\rangle$

SLOCC is a more manageable class of operations

SLOCC

Entanglement classes

Def: $|\phi\rangle$ and $|\psi\rangle$ have the same type of entanglement if $|\phi\rangle \overset{\text{SLOCC}}{\longleftrightarrow} |\psi\rangle$

equivalence relation

$$|\phi\rangle \overset{\text{SLOCC}}{\longleftrightarrow} |\psi\rangle \quad \longleftrightarrow \quad |\psi\rangle = A \otimes B \otimes C |\phi\rangle$$

invertible (A, B, C)

Which are the classes of states of the same type of entanglement ?

SLOCC

Entanglement class= orbit under group

$SL(d) \times SL(d) \times SL(d)$

acting on $C^d \otimes C^d \otimes C^d$

Orbit classification is difficult in general

Has been studied since the 19th century

3 qubit entanglement classes

Dür, Vidal & Cirac

There are six classes, given by representative states

product state $|000\rangle$

biseparable state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$ or permuted

W-state $\frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$

GHZ-state $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

Proof

To show: every 3 qubit state can be transformed to one of the representative state by SLOCC

Case 1: If two ranks equals 1 then state is separable

Case 2: If one rank equals 1 then state is biseparable

Case 3: All ranks equal 2

do Schmidt decomposition of the other two and apply single SL operations with inverse Schmidt coefficients (singular values)

Case 3a: range BC contains two or more product vectors

$\longrightarrow$ state is of GHZ form

Case 3b: range BC contains one product vector

$$|\psi\rangle = |a_1\rangle|b_1\rangle|c_1\rangle + |a_2\rangle|\phi\rangle$$

orthogonal

local unitaries

$$\longrightarrow \quad |a_1'\rangle|0\rangle|0\rangle + |1\rangle|\phi'\rangle \longleftarrow |\phi'\rangle = x|01\rangle + y|10\rangle + z|11\rangle$$

$$t|00\rangle + x|01\rangle + y|10\rangle + z|11\rangle = (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle)$$

$$\longleftrightarrow \quad tz = xy$$

hence there is always a second product vector unless z=0 $\xrightarrow{\text{local SL}}$ W-state

Case 3c: range BC contains no product vectors can easily be shown to not occur