Exercise 1. ***Low-energy soft-sphere scattering I***

In the Born approximation the scattering amplitude reads:

$$f^{(1)}(\theta, \phi) = -\frac{m}{2\pi\hbar^2} \int \mathrm{e}^{i(\vec{k}'-\vec{k})\cdot\vec{r}_0} V(\vec{r}_0)\, d^3\vec{r}_0 \tag{1}$$

For low-energy (long wave-length) scattering the exponential factor is essentially constant in the scattering region and the Born approximation simplifies to:

$$f^{(1)}(\theta, \phi) \approx -\frac{m}{2\pi\hbar^2} \int V(\vec{r}_0)\, d^3\vec{r}_0. \tag{2}$$

Consider the scattering off a soft-sphere potential

$$V(\vec{r}) = \begin{cases} V_0, & \text{if } r \leq a \\ 0, & \text{if } r > a \end{cases}$$

(a) Compute the low-energy scattering amplitude.

(b) Compute the differential cross section.

(c) Compute the total cross section.

Exercise 2. ***Second-order Born approximation***

In the lecture you have seen how to derive the Born approximation starting from the integral representation of the Schroedinger equation.

Extending that reasoning derive a formal expression for the second-order Born approximation.

Exercise 3. ***Low-energy soft-sphere scattering II***

Consider again the scattering off a soft-sphere potential as in exercise 1. Compute the scattering amplitude in the second Born approximation.

Exercise 4. ***One-dimensional Green's function***

Consider the one-dimensional Schroedinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\psi(x) = E\psi(x) \tag{3}$$

Using the Green's function method, show that the integral form of the solution is

$$\psi(x) = \psi_0(x) - \frac{im}{\hbar^2 k}\int_{-\infty}^{\infty} e^{ik|x-y|}V(y)\psi(y)dy \tag{4}$$

where ψ_0 is the solution of the free Schroedinger equation, and $k = \sqrt{2mE/\hbar^2}$.