Proseminar - Modern Topics in Condensed Matter- Quantum Magnetism
R. Chitra, ITP

1 Theme

In this Proseminar, we will focus on the very rich field of Quantum Magnetism. This field essentially explores the physics of interacting localized quantum spins in various spatial dimensions. The standard examples of magnetism are ferromagnets and Neel ordered antiferromagnets where the spins order in a symmetry breaking ground state resulting in interesting collective excitations. However, the interplay of spatial dimensionality, geometric frustration induced by the lattice on which the spins reside, quantum fluctuations and underlying symmetry properties leads to a vast range of novel and fascinating physical phenomena, including quantum phase transitions, quantum criticality, topological order and fractional excitations. These are very complex problems and typically, we use a host of methods ranging from quantum field theoretic methods (bosonization, conformal field theory, large N methods, renormalization group etc) to numerical methods (density matrix renormalization group, exact diagonalization, variational methods) to study these interacting quantum many body systems. The field is also immensely enriched by the plethora of experimental studies on real materials, where a lot of this novel phenomenology is realized. We choose a small subset of these topics, focussing mainly on the physics of antiferromagnets. The students will have the opportunity to study some of the theoretical methodology and apply it to specific problems and where possible also compare theoretical predictions with experimental results. The topics are grouped in various sections. The students are advised to first choose one of the 5 sub-topics presented in this note. The 5th topic looks at frustrated ferromagnets and is a developing field. Though we will focus mainly on the original research literature, it is recommended that the students skim through the books mentioned below to get a better idea about the subject. Books on the topic:

- Interacting Electrons and quantum magnetism, A. Auerbach,
- Quantum Physics in One Dimension, T. Giamarchi
- Field theories of condensed matter physics, E. Fradkin

2 General Remarks

The students give a 40–50 minutes presentation (see Wilkin's rules,

`http://www.physics.ohio-state.edu/~wilkins/`

and select the 'one-pagers' for hints on giving good talks), plus a 20–10 minute chalk-talk presentation (on the blackboard) of some specific (hard) calculation. Take into account that people will ask questions during the talk. Students are supposed to work in collaborative groups of 2–4 members depending on the specific topic. After the presentation, the candidate should answer questions from the audience. The talk should be in English (German only as a special exception)—students are supposed to explain clearly all the physics concepts associated with the problem, including experiments. The talk should show that the topic has been properly understood; during the 10–20 minutes chalk-talk, students are supposed to demonstrate their abilities to perform a difficult calculation and *explain* it properly (intimidation is not a substitute for clarity)—again, students have to prove that they understand what is being done. The time limits will be handled strictly. Each candidate should present either an experiment or a numerical analysis associated with the topic, when relevant.

The original literature (papers) should also be studied. Criteria for obtaining the Credit Points: i) give a presentation, ii) all students, but particularly group members, are expected to ask questions and participate in discussions, iii) be present in at least 80 % of all the sessions,v) hand in a written documentation of your talk, both the transparency part and the chalk-talk part. The documentation has to be written with LaTeX (RevTeX) in a usual paper style (about 6–8 PRB style pages), see Wilkins' rules and ask your assistant. The transparencies and the written reports will be collected in a book and handed out to all the students.

3 Quantum Antiferromagnets

3.1 Order from disorder phenomena

(Accessible for semester students)

The fundamental model to study spin physics is the Heisenberg model. The development of the field of quantum magnetism starts with the understanding of the corresponding classical model, where the quantum spins can be considered as classical vectors [A1]. For instance, standard mean field methods like, spin wave theory etc, stem from a comprehension of the classical ground states [A6]. In the case of many frustrated models, we have a huge degeneracy of the classical ground states. Adding thermal or quantum fluctuations result in a phenomenon called Order from Disorder, wherein fluctuations which typically tend to disorder a system, actually help the system in choosing a non degenerate ground state or a smaller set of degenerate ground states[A2-A6]. The basic papers reviewing this physics:

- [A1] A magnetic analogue of stereoisomerism : application to helimagnetism in two dimensions J.Villain, J.Phys. (Paris) 38, 385 (1977).
- [A2] Order as an effect of disorder, Villain, J., Bidaux, R., Carton, J. P. Conte, R. . J. de Phys. 41 (1980).
- [A3] Ordering due to disorder in a frustrated vector antiferromagnet, Henley, C. L.. Phys. Rev. Lett. 62, 2056 (1989).
- [A4] Order from disorder in a kagome antiferromagnet, Chubukov, A. . Phys. Rev. Lett. 69, 832 (1992).
- [A5] Bosonic mean-field theory for frustrated Heisenberg antiferromagnets in two dimensions, R. Chitra, Sumathi Rao, Diptiman Sen, and S. Suresh Rao Phys. Rev. B 52, 1061(1995).

- [A6] A. Order produced by quantum disorder in the Heisenberg rhombohedral antiferromagnet, Rastelli, E. Tassi, Journal of Physics C, Solid State Physics 20, L303.

3.2 Quantum spin chains

In one dimension, due to enhanced quantum fluctuations, it is well known that quantum systems cannot develop long range order or have local order parameters. However, depending on the details of the system, they can develop quasi-long range order and have gapless excitations, have no order and gapful excitations, develop topological order etc. In one dimensions, one can use a whole host of analytical methods like Bethe Ansatz, bosonization, conformal field theory, exact or variational wave function approaches and the highly accurate numerical methods like the Density Matrix Renormalization Group. This field is further complemented by extensive experimental investigations of a wide range of quasi-one dimensional compounds. Following is a list of relevant papers, books and review articles. The students choosing this subtopic are advised to take a look at the review article *I. Affleck in Fields, Strings and Critical Phenomena, Les Houches, Session XLIX, 1988 (lecture notes)* and then choose a more specialized topic. The review and the books are comprehensive with a lot of relevant technical detail. Specific seminar topics will include applying some of the methodology (Jordan Wigner transformations, Bethe Ansatz, bosonization (abelian and non-abelian), mapping to conformal field theories, renormalization group, density matrix renormalization group etc) to different physical problems and comparing the theoretical predictions with experiments (when available).

- [B1] Bethe Ansatz- Antiferromagnetic Heisenberg spin chains, H.A. Bethe, Z. Physik 71, 205 (1931).
- [B2] Critical theory of quantum spin chains, I. Affleck and F.D.M. Haldane, Phys. Rev. B 36, 52915300 (1987);
- [B3] Non-linear Field Theory of a Frustrated Heisenberg Spin Chain, S. Rao and D. Sen, Nucl.Phys. B424 (1994) 547-566.
- [B4] Quasi-one dimensional spin half chains (Bosonization and sine Gordon theories) D. Cabra, Field Theoretical methods in Quantum Magnetism, Chapter 6 Lecture Notes in Physics Volume 645, 2004, pp 253-305.
- [B5] Rigorous results on valence-bond ground states in antiferromagnets, I. Affleck, Kennedy, Lieb and Tasaki Phys. Rev. Lett 59, 799 (1987).
- [B6] Quantum spin chains and the Haldane gap, I. Affleck J. Phys. Condens. Matter 1 3047 (1989).
- [B7] Hidden symmetry breaking and the Haldane phase in S=1 quantum spin chains, Tom Kennedy and Hal Tasaki, Communications in Mathematical Physics 147, 431 (1992).
- [B8] Critical properties of gapped spin-2 chains and ladders in a magnetic field R. Chitra and T. Giamarchi, Phys. Rev. B 55, 58165826 (1997).

- [B9] The density-matrix renormalization group, Ulrich Schollwoeck, Rev. Mod. Phys. 77, 259 (2005).

Some experimental papers:

- [B7] Magnetic Susceptibility of Ideal Spin 1 /2 Heisenberg Antiferromagnetic Chain Systems, Sr2CuO3 and SrCuO2, N. Motoyama, H. Eisaki, and S. Uchida ,Phys. Rev. Lett. 76, 32123215 (1996).
- [B8] Fractional spinon excitations in the quantum Heisenberg antiferromagnetic chain Martin Mourigal, Mechthild Enderle, Axel Klpperpieper, Jean-Sbastien Caux, Anne Stunault and Henrik M. Rnnow Nature Physics 9, 435441 (2013).
- [B9] Spin gap in a quasi-one-dimensional S = 1/2 antiferromagnet: Cu2Cl4, P. R. Hammar, D. H. Reich, C. Broholm, and F. Trouw Phys. Rev. B 57, 7846 (1998).
- [B10] NMR Imaging of the Staggered Magnetization in the Doped Haldane Chain Y2BaNi1-xMgxO5, F. Tedoldi, R. Santachiara and M. Horvati?, Phys. Rev. Lett. 83, 412415 (1999).

3.3 Two dimensional spin systems

In two dimensions, the quantum spins can in principle order at zero temperature. This happens for instance in the Heisenberg antiferromagnet on a square lattice. However, lattice geometry can induce frustration between the spins, which depending on the degree can result in completely disordered spin liquid phases. There are different kinds of spin liquids which have non trivial features depending symmetries and lattice geometries. We take a peek at this highly complex spin physics which is the subject of a lot of current research. Students choosing this topic are advised to peruse the overview article *Quantum spin liquids by F. Mila,Eur. J. Phys. 21 499 (2000)* before choosing their specific topic. [C2, C5, C8, C9, C10] are review articles which focus on various aspects of the spin physics in two dimensions.

- [C1] Heisenberg ferro and antiferromagnets on square lattices- Schwinger Bosons Approach to Quantum Magnetism, Arovas and Auerbach, Lecture Notes arxiv:0809.4836.
- [C2] The spin-1/2 Heisenberg antiferromagnet on a square lattice and its application to the cuprous oxides, E. Manousakis Rev. Mod. Phys. 63, 1 (1991).
- [C3] Ground states of a frustrated spin-1 antiferromagnet: Cs2CuCl4 in a magnetic field, M. Y. Veillette,1 J. T. Chalker,1 and R. Coldea, Physical Review B 71, 214426 (2005).
- [C4] Large- N expansion for frustrated quantum antiferromagnets Read, N. and Sachdev, S., Phys. Rev. Lett. 66, 1773 (1991).
- [C5] Mean-field theories of spin liquids and topological orders - X. G. Wen, Cargese summer school 1990; Phys. Rev. B 44, 2664 (1991).

- [C6] Strong geometric frustration- Quantum Kagome Antiferromagnet, Asakawa and Suzuki, International Journal of Modern Phys B 9, 933 (1995).
- [C7] Spin-Liquid Ground State of the S = 1/2 Kagome Heisenberg Antiferromagnet Simeng Yan, David A. Huse, Steven R. White,Science 332, 1173-1176 (2011).
- [C8] Strongly frustrated spin liquids - Insight article Leon Balents, Nature 464, 199-208 (11 March 2010).
- [C9] G. Misguich, "Quantum spin liquids" arxiv:0809.2257 .
- [C10] Symmetries and fractionalization in spin liquids - G. Misguich, Springer Series in Solid-State Sciences Volume 164, pp 407-435 (2011).
- [C11] Quantum dimer models, Moessner and Raman arxiv:0809.3051 (2008)- lecture notes.

Experimental literature:

- [C11] Spin-1/2 kagome compounds: Volborthite vs Herbertsmithite,Z Hiroi , H Yoshida, Y Okamoto and M Takigawa J. Phys.: Conf. Ser. 145 012002 (2009).
- [C12] Fractionalized excitations in the spin liquid state of a kagom lattice antiferromagnet, Han et al, Nature 492, 406-410 (2012).

3.4 Topological aspects

This section deals with more recent developments on the notion of topological phases in interacting spin systems. Some spin systems like transverse field Ising chain and the Kitaev model have non trivial topological phases which have localized Majorana fermion modes and further support abelian and non-abelian anyonic excitations. These further lead to questions about the methodology required to address topological aspects in interacting many body systems.

- [D1] Anyons in an exactly solved model and beyond, Annals of Physics 321 (2006) 2.
- [D2] Topological Entanglement Entropy, Kitaev and Preskill, Phys.Rev.Lett. 96 (2006) 110404.
- [D3] Topological phases and quantum computation, Kitaev and Laumann, Les Houches 2008, arXiv:0904.2771.
- [D4] Entanglement Entropy as a Portal to the Physics of Quantum Spin Liquids, Tarun Grover, Yi Zhang and Ashvin Vishwanath, New J. Phys. 15, 025002 (2013).

4 Spin Ice

Spin ices are the new frontier in magnetism. As opposed to the geometric frustration in antiferromagnetic systems, frustration can also arise in ferromagnetically coupled spins under certain conditions. In some cases, this results in the system not having any well defined minimal energy state and a residual entropy. These are the spin ice systems seen in pyrochlore compounds whose physics resembles that of water ice. These systems have been shown to possess deconfined magnetic monopole excitations. Students who choose this topic are expected to review different aspects of the field.

- [E1] Spin Ice, M. Gingras, arXiv:0903.2772, (2009).
- [E2]Spin Ice, Fractionalization, and Topological Order, C. Castelnovo, R. Moessner, and S.L. Sondhi, Annual Review of Condensed Matter Physics Vol. 3: 35-55 (2012).
- [E3] Dirac Strings and Magnetic Monopoles in Spin Ice Dy2Ti2O7 D.J.P. Morris et al, Science 326 no. 5951 pp. 411-414 (2009) .

The following tutors will assist you: A. Benlagra, Oded Zilberberg, A. Lebedev, S. Schmidt, Evert van , C. Candu, M. Baggio, C. Peng, L. Wang and M. Iazzo.