Fourth Exercise Sheet due to 27. March

The purpose of this exercise is to get familiar with the Jaynes-Cummings model describing an interaction of a single cavity mode (harmonic oscillator) with a two level atom. The Hamiltonian is

$$\begin{aligned} H &= H_0 + H_{int} \\ &= \omega_c(a^*a + \frac{1}{2}) + \frac{\omega_0}{2}\sigma_z + \frac{\Omega}{2}[a\sigma_+ + a^*\sigma_-], \end{aligned}$$

where $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$ and a is the annihilation operator. The free Hamiltonian H_0 has eigenvectors $|n, \pm\rangle$,

$$H_0|n, \pm\rangle = (\omega_c(n + 1/2) \pm \frac{\omega_0}{2})|n, \pm\rangle.$$

The model exhibits different behavior depending on the value of the detuning $\delta = \omega_c - \omega_0$.

Exercise 1 (The resonant case, $\delta = 0$) Show that in the resonant case $[H_0, H_{int}] = 0$. Use this to compute the time evolution of an initial state $|0, +\rangle$. It exhibits Bloch oscillations between $|0, +\rangle$ and $|1, -\rangle$, periodically absorbing and emitting a photon.

Exercise 2 (The non-resonant case) Suppose that $\delta/\Omega >> 1$. Use second order perturbation theory to derive eigenvalues of H. You shall see that there is no correction in the first order and a non-zero correction in the second order. We shall use this result in the class but we will not derive it there.