Eight Exercise Sheet due to 8. May

Exercise 1 (Examples of the weak coupling limit) *We consider a two level system coupled to a bath of harmonic oscillators with frequencies ranging from* 0 *to a UV cut off. Compute the Lindblad generator of the dynamics in the weak coupling limit (*$\lambda \to 0$*) for the following two cases. Express the results in terms of the mean number of excitations* $n_\omega = \mathrm{Tr}(a^*_\omega a_\omega G)$*, where* G *is the initial state of the bath.*

a)

$$H = \frac{\omega_0}{2}\sigma_z + \int_0^{cut\,off} \omega a^*_\omega a_\omega \mathrm{d}\omega + \lambda \int_0^\infty h(\omega)\sigma_x(a_\omega + a^*_\omega)$$

b)

$$H = \frac{\omega_0}{2}\sigma_z + \int_0^{cut\,off} \omega a^*_\omega a_\omega \mathrm{d}\omega + \lambda \int_0^\infty h(\omega)\sigma_z(a_\omega + a^*_\omega)$$

In this case the free Hamiltonian commutes with the interaction (dephasing). This is somehow singular and you shall see that you can get meaningful answer only if you assume that $\lim_{\omega\to 0} h(\omega) = 0$.