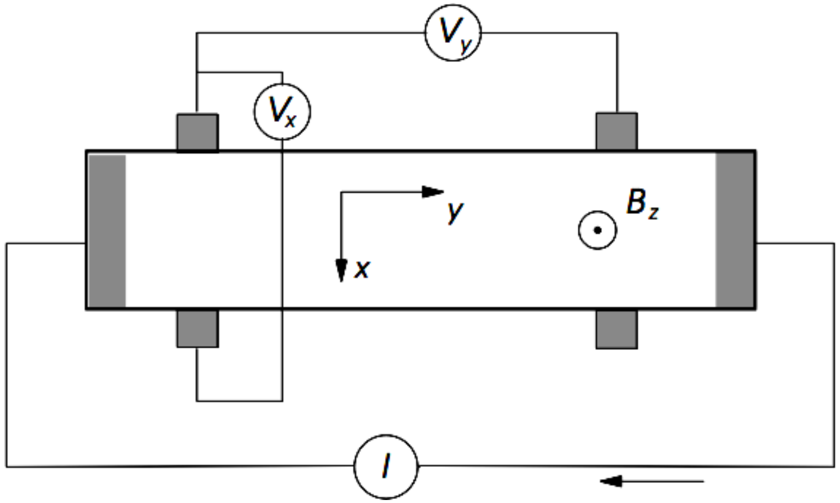
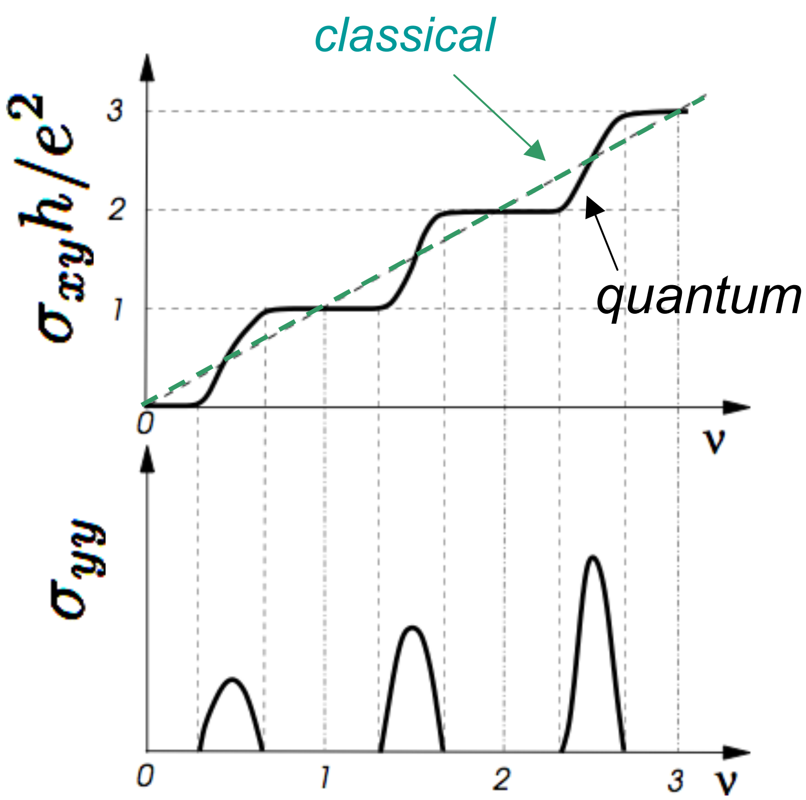
# Quantum Hall effect integer

## Hall bar geometry




## classical Hall effect

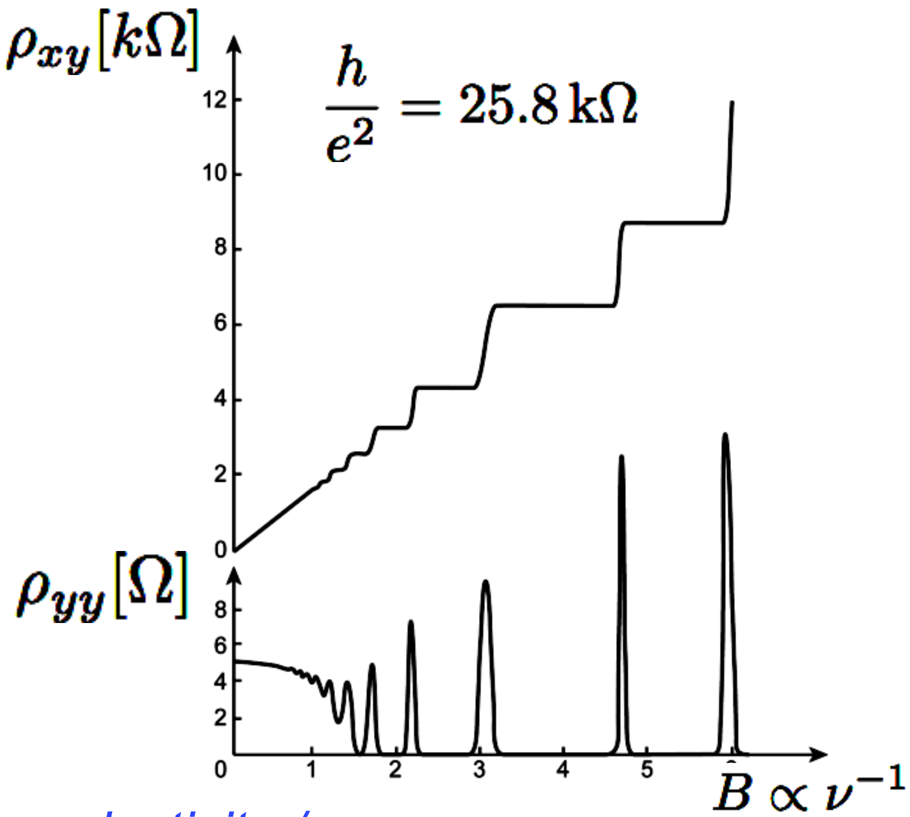
$$\sigma_H = \sigma_{yx} = \frac{j_y}{E_x} = \nu \frac{e^2}{h}$$

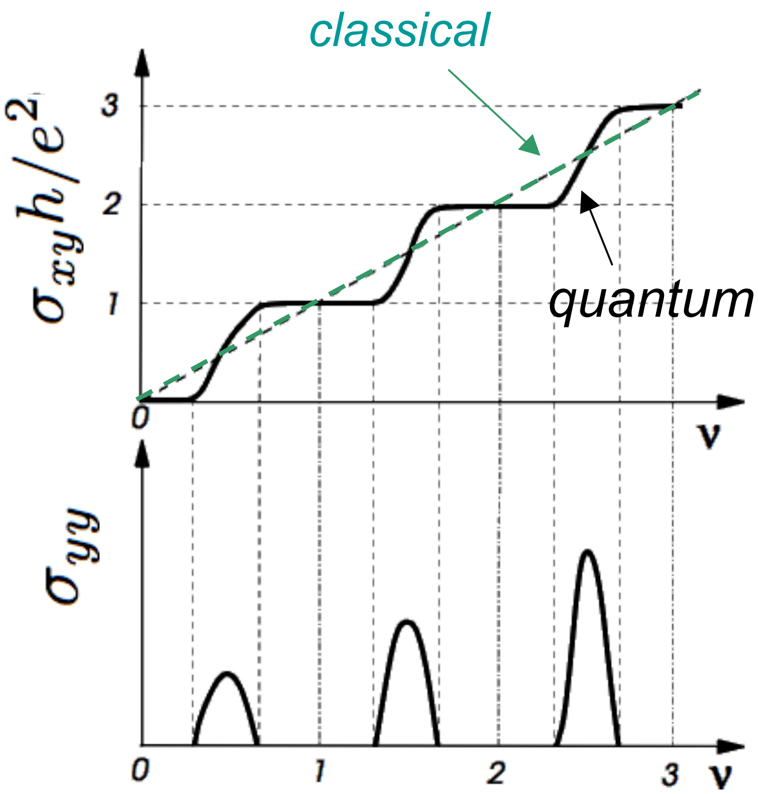
$$\Phi_0 = \frac{hc}{e}$$ flux quantum

$$\Phi_e = \frac{B}{n_0}$$ flux per electron

$$\nu = \frac{n_0 hc}{eB} = \frac{\Phi_0}{\Phi_e} = \frac{1}{\text{flux quanta per electron}}$$

# Quantum Hall effect integer




conductivity /
resistivity tensor

$$\hat{\sigma} = \hat{\rho}^{-1}$$

$$\sigma_{yy} = \frac{\rho_{yy}}{\rho_{yy}^2 + \rho_{xy}^2}$$

$$\sigma_{xy} = \frac{\rho_{xy}}{\rho_{yy}^2 + \rho_{xy}^2}$$

$$\rho_{xy} \neq 0$$

$$\rho_{yy} = 0 \quad \Leftrightarrow \quad \sigma_{yy} = 0$$

# Quantum Hall effect    integer

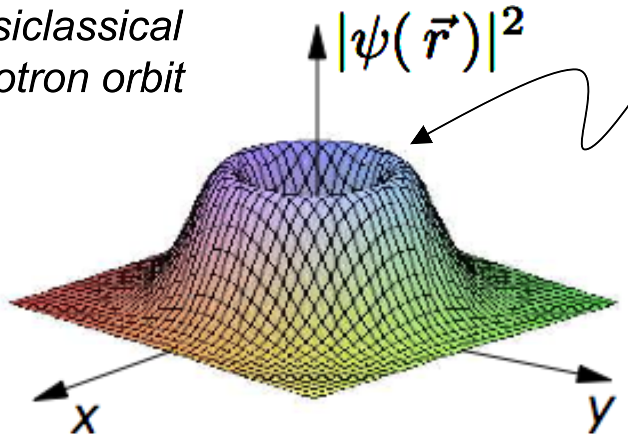
**Landau levels:** *symmetric gauge* $\vec{A} = \frac{B}{2}(-y, x, 0)$ $\quad \vec{B} = (0, 0, B)$

$$\frac{\hbar^2}{2m^*}\left[-\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \left(\frac{1}{r}\frac{\partial}{\partial \varphi} - i\frac{e}{2\hbar c}Br\right)^2\right]\psi(r,\varphi) = E\psi(r,\varphi)$$

ground state $\quad E_{n=0} = \hbar\omega_c/2 \quad \omega_c = |eB|/m^*c \quad \ell^2 = \hbar c/|eB|$

$$\psi_{n=0,m}(r,\varphi) = \frac{1}{\sqrt{2\pi\ell^2 2^m m!}}\left(\frac{r}{\ell}\right)^m e^{-im\varphi}e^{-r^2/4\ell^2} \qquad m = 0, 1, 2, \ldots$$

*quasiclassical cyclotron orbit*




radius (peak): $r_m = \sqrt{2m}\ell$

$$\pi B r_m^2 = 2\pi m \ell^2 B = 2\pi m \frac{B\hbar c}{eB} = m\Phi_0$$

integer flux quantum enclosed

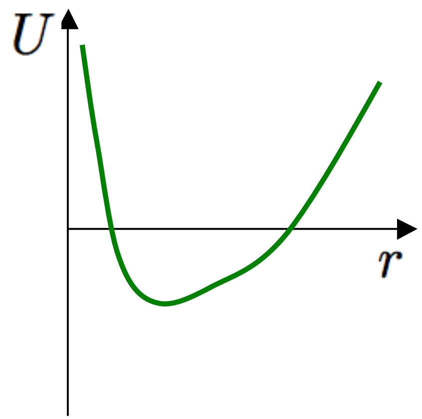
# Quantum Hall effect integer

## Landau levels & circular potential

$$\frac{\hbar^2}{2m^*}\left[-\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}-\left(\frac{1}{r}\frac{\partial}{\partial \varphi}-i\frac{e}{2\hbar c}Br\right)^2\right]\psi(r,\varphi)+U(x,y)\psi(r,\varphi)=E\psi(r,\varphi)$$

$$U(x,y)=U(r)=\frac{C_1}{r^2}+C_2r^2+C_3$$

exactly solvable




$$\tilde{\psi}_{0,m}(r,\varphi)=\frac{1}{\sqrt{2\pi\ell^{*2}2^{\alpha}\Gamma(\alpha+1)}}\left(\frac{r}{\ell^*}\right)^{\alpha}e^{-im\varphi}e^{-r^2/4\ell^{*2}}$$

$$m=0,1,2,\ldots$$

$$\alpha^2=m^2+C_1^*$$

$$\ell^{*2}=\frac{\ell^2}{\sqrt{1+C_2^*}}$$

$$C_1^*=\frac{2m^*C_1}{\hbar^2}\qquad C_2^*=\frac{8\ell^4m^*C_2}{\hbar^2}$$

lifting the degeneracy

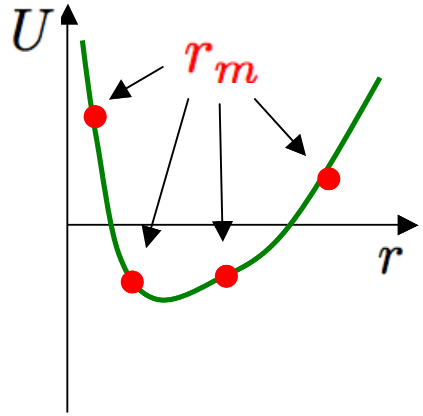
$$E_{0,m}=\frac{\hbar\omega_c}{2}\left[\frac{\ell^2}{\ell^{*2}}(\alpha+1)-m\right]+C_3$$

$$r_m=\sqrt{2\alpha}\ell^*$$ wave function peak position

# Quantum Hall effect integer

## Landau levels & circular potential

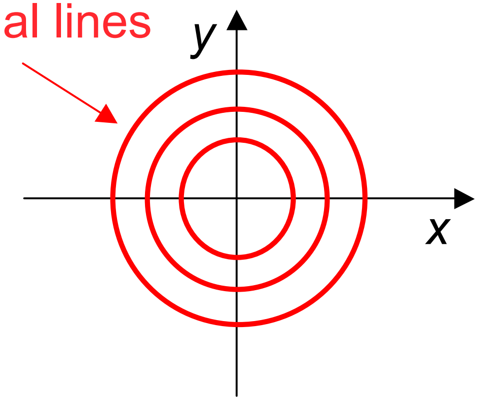
$$U(x,y) = U(r) = \frac{C_1}{r^2} + C_2 r^2 + C_3$$



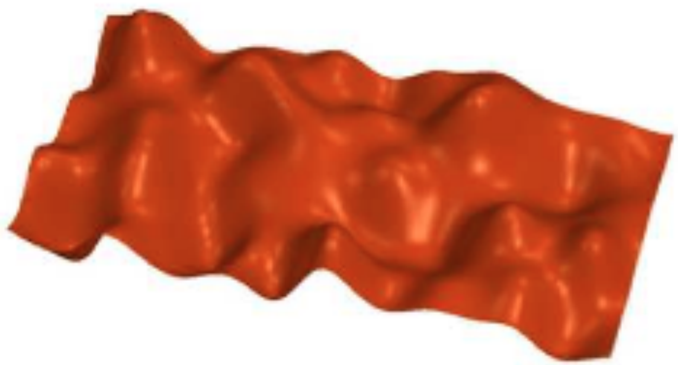

quasiclassical limit $C_1^*, C_2^* \ll 1$ (weak potential)

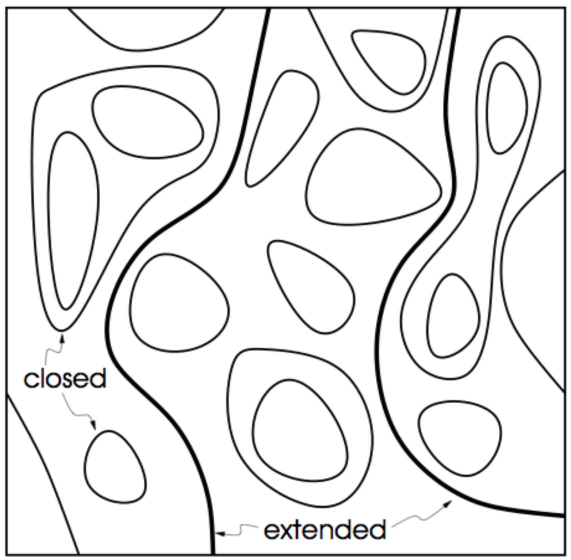
$$E_{0,m} \approx \frac{\hbar\omega_c}{2} + \frac{C_1}{r_m^2} + C_2 r_m^2 + C_3 + \ldots$$

trajectories
equipotential lines




random potential landscape

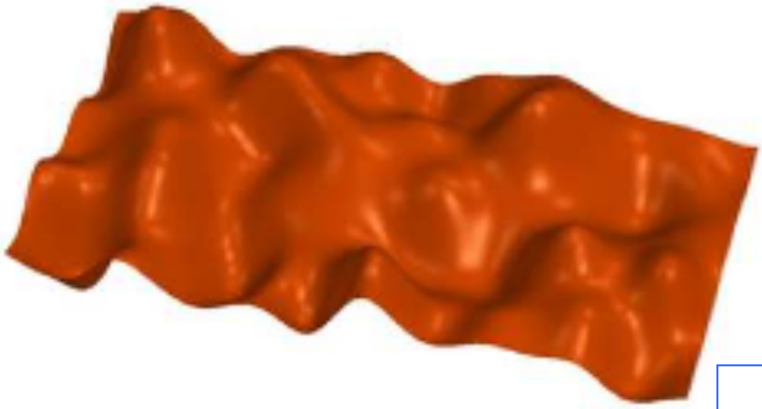




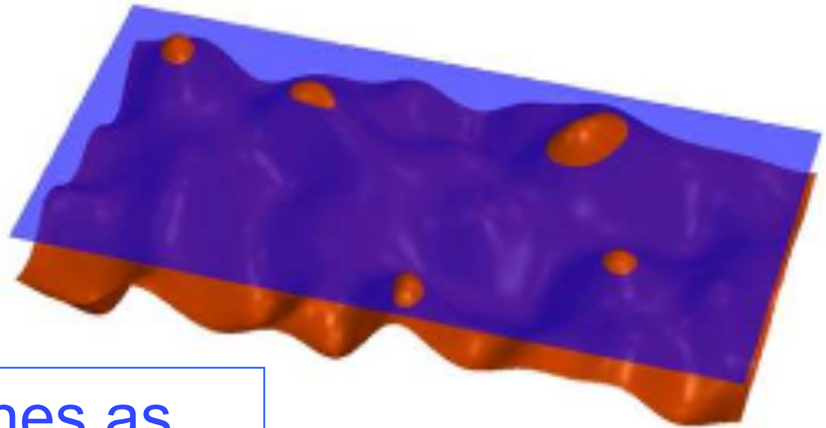

closed and
extended
trajectories

# Quantum Hall effect integer

Potential landscape

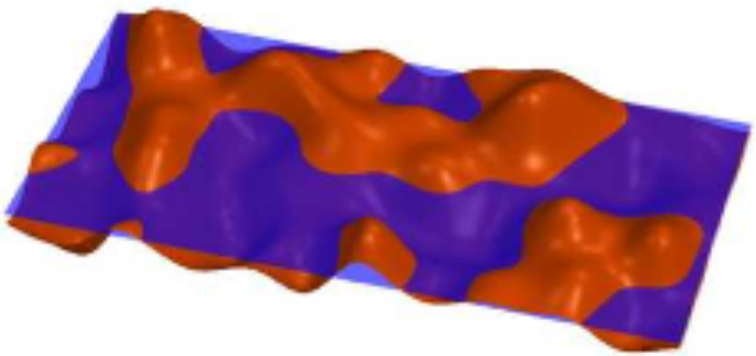


islands

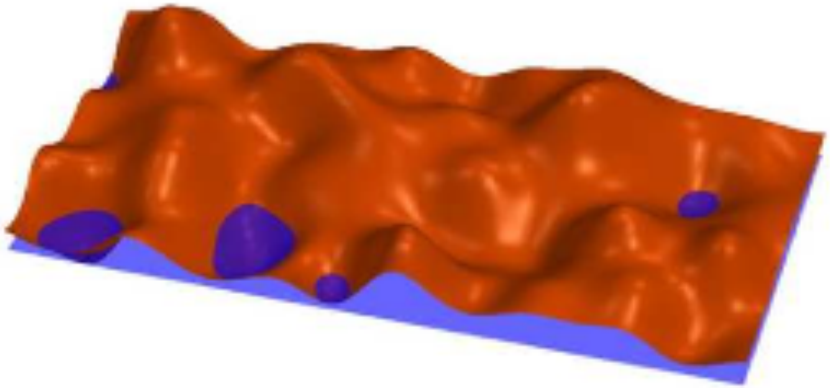


coastlines as
equipotential lines
(contour lines)

percolating coastlines

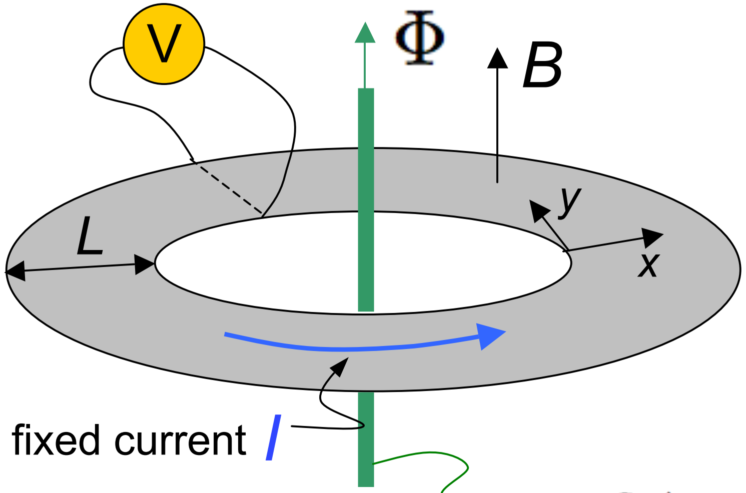


lakes



# Quantum Hall effect integer

## Laughlins argument




- uniform magnetic field B through Corbino disk
- change of Aharonov-Bohm phase through $\Phi \rightarrow \Phi + \delta\Phi$
- Aharonov-Bohm phase acts on extended trajectories around the Corbino ring

$$\delta A_\varphi = \frac{\delta\Phi}{2\pi r} \quad \begin{cases} \vec{A} \rightarrow \vec{A} + \delta\vec{A} = \vec{A} + \vec{\nabla}\chi \\ \psi \rightarrow \psi\, e^{ie\chi/\hbar c} = \psi\, e^{i(\delta\Phi/\Phi_0)\,\varphi} \end{cases}$$

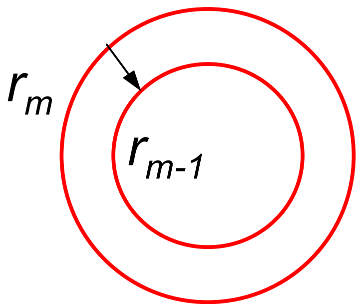
### single-valued wave function

$$e^{-im\phi} \rightarrow e^{-i(m-\delta\Phi/\Phi_0)\phi}$$

$$m \rightarrow m - \delta\Phi/\Phi_0$$ *integer*

$$B\pi r_m^2 = m\Phi_0 + \delta\Phi$$ conserved flux enclosed by trajectory

### extended state (pure case)



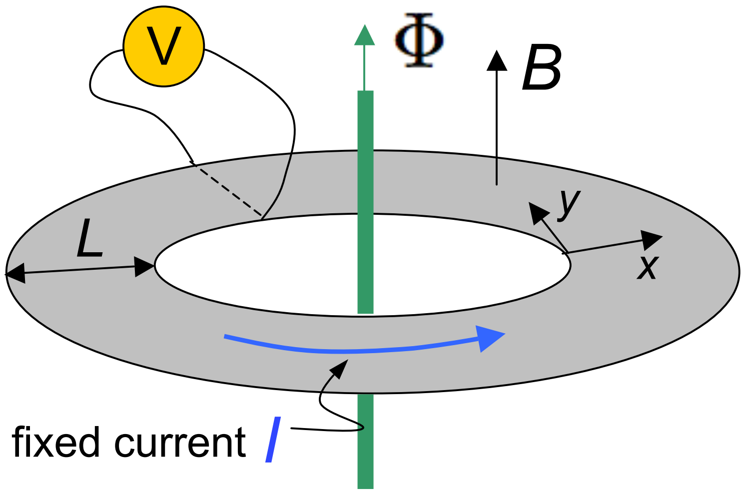

shift by one "trajectory"

$$\delta\Phi \rightarrow \Phi_0$$

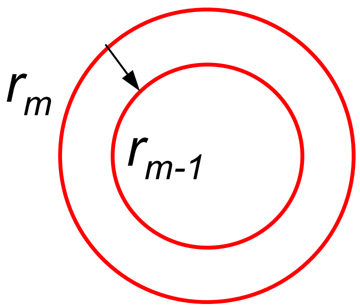
$$m \rightarrow m - 1$$

# Quantum Hall effect integer

## Laughlins argument




$$m \to m - \delta\Phi/\Phi_0$$




shift by one "trajectory"

$$\delta\Phi \to \Phi_0$$

$$m \to m - 1$$

## energy argument $\delta\Phi \to \Phi_0$

→ net shift of 1 el from outer to inner edge

- potential energy

  moving electron against electric potential (V)

  $$\Delta\epsilon_V = -eE_x L$$

- electromagnetic energy

  inductive energy of current loop

  $$\Delta\epsilon_I = I_y\,\delta\Phi/c$$

no energy change for $\delta\Phi \to \Phi_0$

gauge invariance $\Delta\epsilon_I + \Delta\epsilon_V = 0$

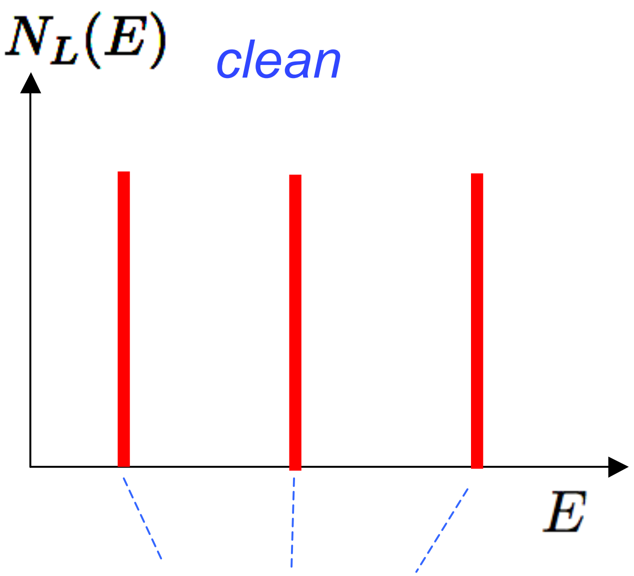
*per filled Landau level* $\sigma_H = \dfrac{j_y}{E_x} = \dfrac{I_y}{LE_x} = \dfrac{e^2}{h}$

# Quantum Hall effect — integer

## localized versus extended states

Landau levels (without spin) $$E_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

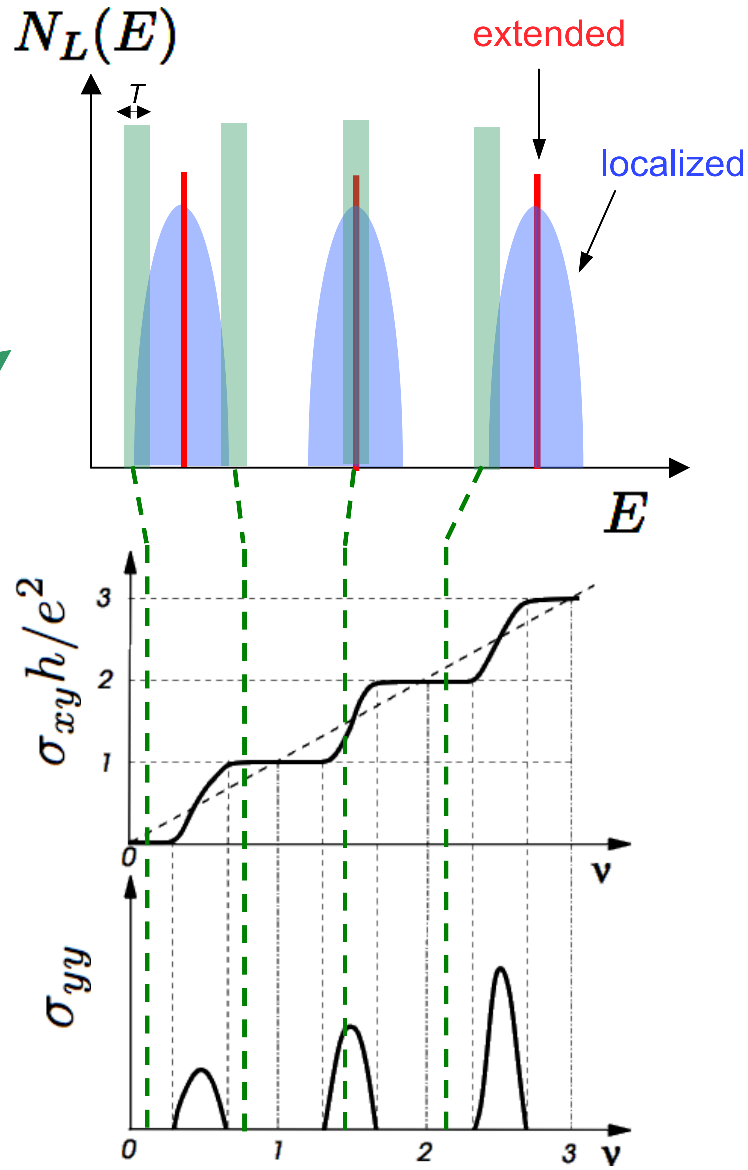
$$N_L(E) = \frac{L_x L_y}{2\pi\ell^2}\sum_n \delta(E - E_n)$$




$$\frac{L_x L_y}{2\pi\ell^2} = \frac{L_x L_y |B|}{\Phi_0} = \frac{N}{\nu}$$

degeneracy

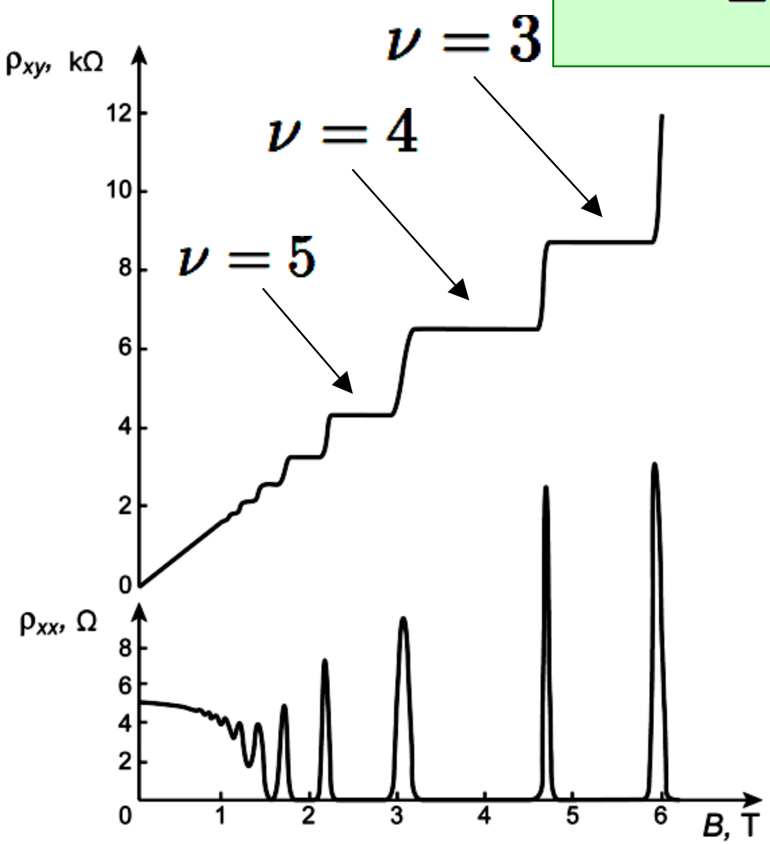

potential landscape lifts degeneracy



# Quantum Hall effect fractional

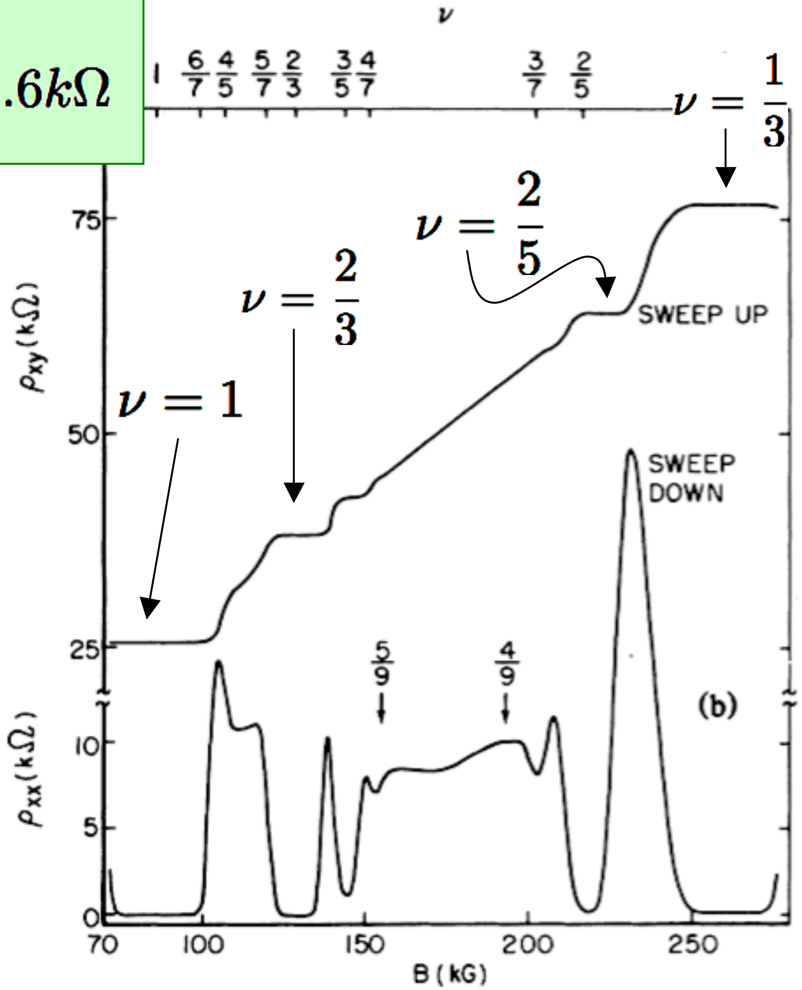
$$\rho_{xy} = \frac{h}{e^2\nu} = \frac{1}{\nu} \times 25.6k\Omega$$

integer QHE




von Klitzing, Dorda and Pepper (1980)

fractional QHE




Störmer, Tsui and Gossard (1982)

# Properties of metals

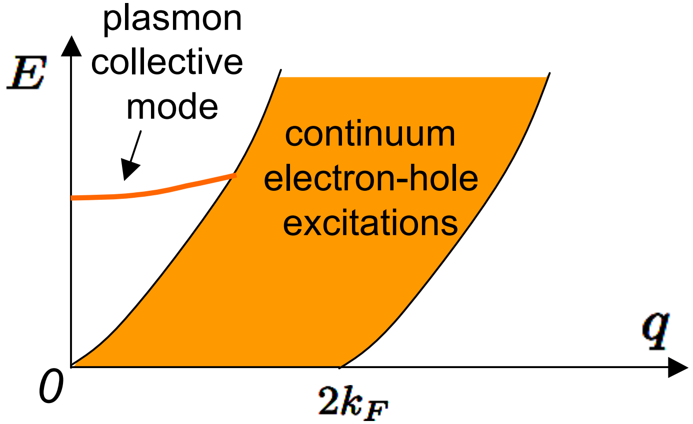
## properties of metal

- well described by "free electrons"
  Jellium-model (lattice not essential)

- strong renormalization of external perturbations:
  dynamical dielectric function

$$V(\vec{q},\omega) = \frac{V_a(\vec{q},\omega)}{\varepsilon(\vec{q},\omega)}$$

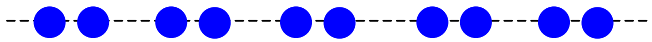
## elementary excitations




## novel phases

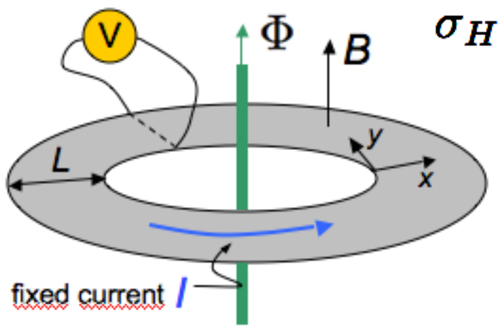
- Fermi surface instability
  e.g. Peierls instability
  *metal* → *insulator*

  Charge Density Wave



interaction-driven
spontaneous symmetry breaking

- Quantum Hall effect




$$\sigma_H = \nu \frac{e^2}{h}$$

topological phase