band filling

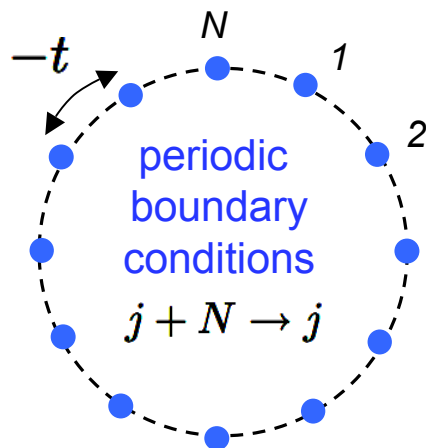## tight-binding hopping model

$$\mathcal{H} = -t \sum_{j=1}^{N} \sum_{s=\uparrow,\downarrow} \{\hat{c}_{j+1,s}^{\dagger} \hat{c}_{j,s} + \hat{c}_{j,s}^{\dagger} \hat{c}_{j+1,s}\}$$

$$\hat{c}_{j,s} = \frac{1}{\sqrt{N}} \sum_{k} \hat{a}_{k,s} e^{iR_j k} \qquad R_j = ja$$

$$\mathcal{H} = \sum_{k,s} \epsilon_k \hat{a}_{k,s}^{\dagger} \hat{a}_{k,s} \qquad \epsilon_k = -2t \cos ka$$
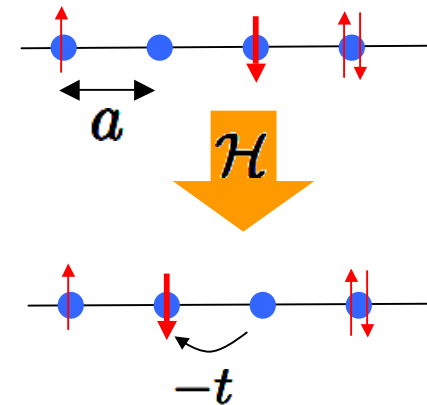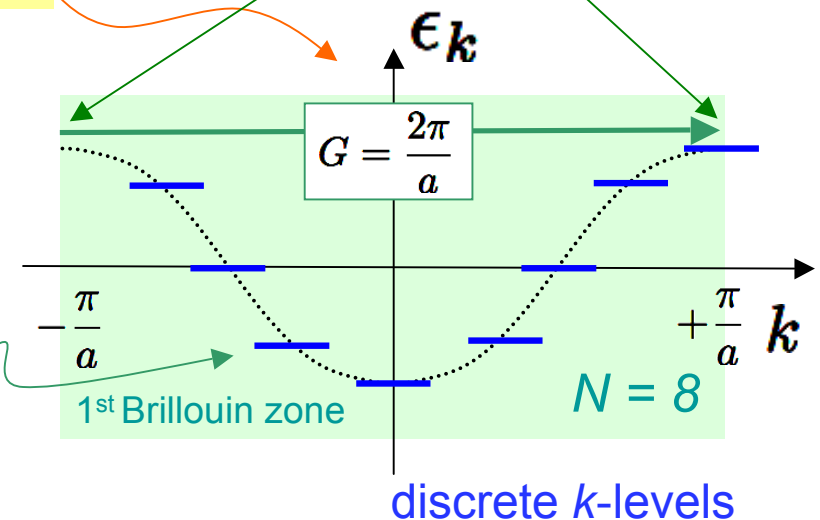
system with N sites

$-t$ N   1

periodic
boundary
conditions
$j + N \to j$

$$e^{iR_j k} = e^{i(R_j + L)k}$$

$$Lk = Nak = 2\pi n$$

$$k = \frac{2\pi}{L} n = \frac{2\pi}{a} \frac{n}{N}$$

a

$\mathcal{H}$

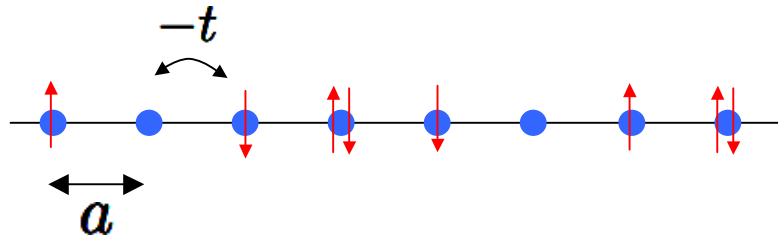$-t$

note:
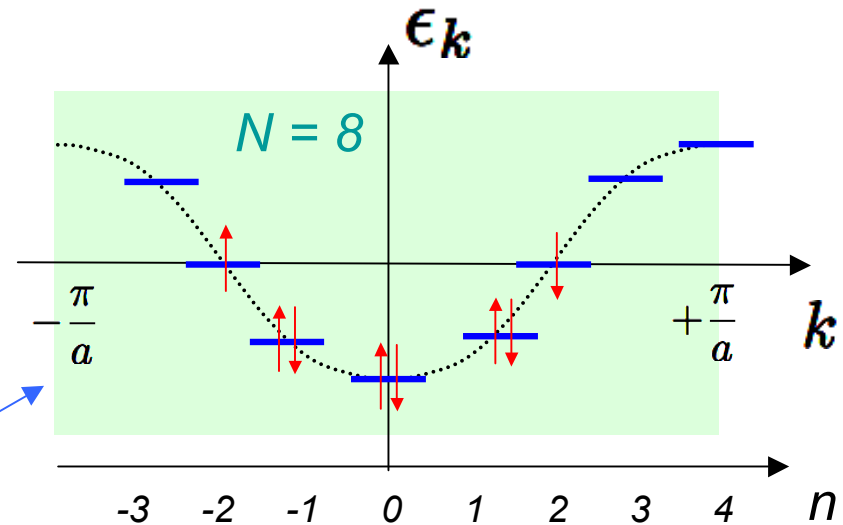$$k + G \to k$$

ϵ_k

$$G = \frac{2\pi}{a}$$

$-\frac{\pi}{a}$ $+\frac{\pi}{a}$ k

$N = 8$

1st Brillouin zone

discrete k-levels

band filling

half filled band

N electrons

$$-t$$

$$a$$

ground state
occupy lowest energy levels

ϵ_k

N = 8

$-\dfrac{\pi}{a}$ $+\dfrac{\pi}{a}$ k

-3   -2   -1   0   1   2   3   4   *n*

$$k = \frac{2\pi}{L}n = \frac{2\pi}{a}\frac{n}{N}$$

completely filled band

2N electrons

ϵ_k

N = 8

$-\dfrac{\pi}{a}$ $+\dfrac{\pi}{a}$ k