Sheet 5

Due date: 28 March 2014

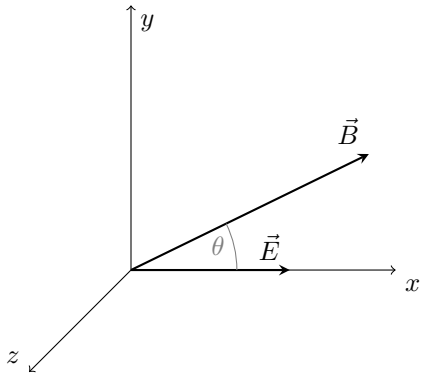
Exercise 1 [*Car paradox*]:

(i) A car of length 4m enters at high velocity corresponding to a Lorentz factor of $\gamma = 2$, a garage of length 3m. The garage has two gates. Let us first consider the situation in which both gates are open, and the car simply passes through the garage. Then, from the point of view of an observer in the reference frame of the garage (the garage keeper), the car is 2m long, and hence that observer believes that the car fits (at a certain moment in time) into the garage. On the other hand, from the point of view of the car driver the garage has length 1.5 m, and thus he claims that the car can never fit into the garage. Explain why this is not a paradox.

(ii) Let us consider the situation in which the gate at the end of the garage is closed. From the point of view of the garage keeper, the car is fully contained in the garage at time t_0, at which moment it has still 1m space at the front. At time t_0 the garage keeper switches on a braking mechanism that acts both on the front and back wheels of the car. At the same time he closes the back gate of the garage. The car driver, on the other hand, fears that at time t_0 when the braking mechanism is initiated, the car still protrudes by 2.5 m from the garage and that as a consequence the back of his car will be destroyed. Explain why his fears are unjustified.

(iii) However, the car driver still has to worry since the car will eventually need to be compressed to at least 3m by the above braking manoeuver. Why is this so? Explain how the braking manoeuver should have been done (without any gates) in order to avoid this compression of the car.

Exercise 2 [*Lorentz transformation of the electromagnetic field*]: In a certain reference frame a static, uniform, electric field $\mathbf{E}_0$ is parallel to the x-axis, and a static, uniform, magnetic field $|\mathbf{B}_0| = 2|\mathbf{E}_0|$ lies in the x-y plane, enclosing an angle θ with the x-axis.



Determine the relative velocity of a reference frame in which the electric and magnetic fields are parallel. What are the fields in that frame for $\theta \to 0$ and $\theta \to (\pi/2)$?