Extra Sheet

`Due date: 8 April 2014 (only discussion)`

Exercise 1 [*Minimal surface of revolution*]: We generate a surface of revolution by rotating a curve with fixed endpoints (x_1, y_1) and (x_2, y_2) around the y-axis (fig. 1), where $0 < x_1 < x_2$ and $y_1 < y_2$. Determine the curve $y(x)$ that minimises the area.

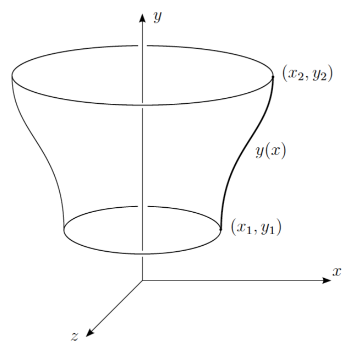

Figure 1: Surface of revolution around the y-axis

Hint: First compute the area A of the surface as a function of the curve $y(x)$, then apply the extremal principle to the functional $A[y(x)]$. As the problem is invariant under translation in the y direction, solving the Euler-Lagrange equation is simplified by the corresponding conserved quantity.

Exercise 2 [*Simple systems*]: Find the Lagrangian and the corresponding equations of motion for the following systems:

(i) **the pendulum in the plane:** First identify the constraints of the system depicted in fig. 2 and find a good choice of independent parameters that describe the possible configurations of the system. Write the kinetic T and potential V energy in terms of these parameters, and define $L = T - V$. Then write down the Euler-Lagrange equations for this Lagrange function. Compare the resulting equations of motion with what you would have obtained by the traditional approach, using forces and constraints.

(ii) **the double pendulum in the plane:** Do a similar analysis with the planar double pendulum as shown in fig. 3.

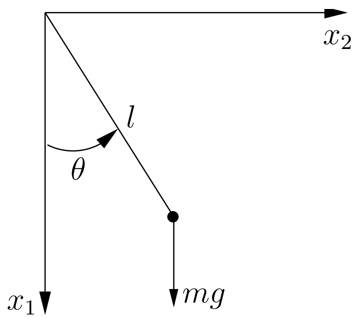

Figure 2: Single pendulum in the plane

Hint: Use the trigonometric identity

$$\cos\phi_1\cos\phi_2 + \sin\phi_1\sin\phi_2 = \cos(\phi_1 - \phi_2)\ .$$

In this case it may be difficult to do the traditional analysis of forces and constraints explicitly.

(iii) [optional] **the centrifugal governor:** Do a similar analysis for the centrigual governor (fig. 3), and determine the equilibrium solutions ($\theta = \text{const}$).

The centrifugal governor consists of two massless rods of length a which are attached to an axis rotating at an angular velocity Ω. At the end of the rods, there are two identical masses m_1. These are connected to two further massless rods of length a by joints. At their lower ends, these rods are connected to a slider of mass m_2 which can move along the z-axis without friction.

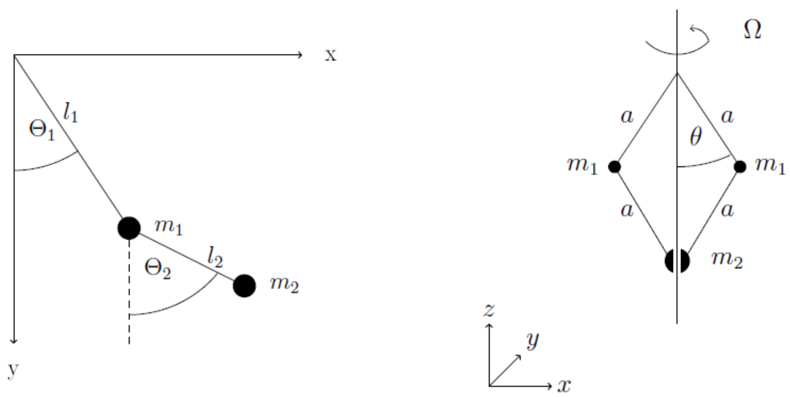

Figure 3: Double pendulum and centrifugal governor