Exercise Sheet III

Hand in by 22.10.2008

Problem 1 [*Open strings ending on D-branes of various dimensions.*]: Consider a world with d spacial dimensions. A Dp-brane is an extended object with p spacial dimensions: a p-dimensional hyperplane inside the d-dimensional space. We will examine properties of strings ending on a Dp-brane, where $0 \leq p < d$. The case $p = d$ where the D-brane is space filling is just the free string.

For a Dp-brane, let x^i, with $i = 1, \ldots, p$, correspond to directions on the Dp-brane and x^a with $a = p+1, \ldots, d$, correspond to directions orthogonal to the Dp-brane. The Dp-brane position is specified by $x^a = 0$, for $a = p+1, \ldots, d$. Open string endpoints must lie on the Dp-brane. Focusing on the $\sigma = 0$ endpoint, we thus have

$$X^a(t, \sigma = 0) = 0, \qquad\qquad a = p+1, \ldots, d.$$

The motion of the endpoint along the D-brane directions x^i is free. We work in the static gauge.

(a) State the conditions satisfied by $\mathcal{P}^\sigma_0$, $\mathcal{P}^\sigma_i$ and P^σ_a at the endpoint (no condition is a possibility!).

Prove that:

(b) All boundary conditions are automatically satisfied if the string ends on a D0-brane.

(c) For a string ending on a D1-brane, the tangent to the string at the endpoint is orthogonal to the D1-brane, and the endpoint velocity is unconstrained.

(d) For a string ending on a Dp-brane, with $p \geq 2$, there are two possibilities:

  (i) the string is orthogonal to the Dp-brane at the endpoint, and the endpoint velocity is constrained, or,

  (ii) the string is not orthogonal to the Dp-brane at the endpoint, and the endpoint moves with the speed of light transversely to the string.

Problem 2 [*Kasey's relativistic jumping rope.*]: Consider a relativistic open string with fixed endpoints:

$$\vec{X}(t, 0) = \vec{x}_1, \qquad\qquad \vec{X}(t, \sigma_1) = x_2.$$

The boundary condition at $\sigma = 0$ is satisfied by the following solution of the wave equation:

$$\vec{X}(t, \sigma) = \vec{x}_1 + \frac{1}{2}\left(\vec{F}(ct + \sigma) - \vec{F}(ct - \sigma)\right). \tag{1}$$

Here $\vec{F}$ is a vector valued function of a single variable.

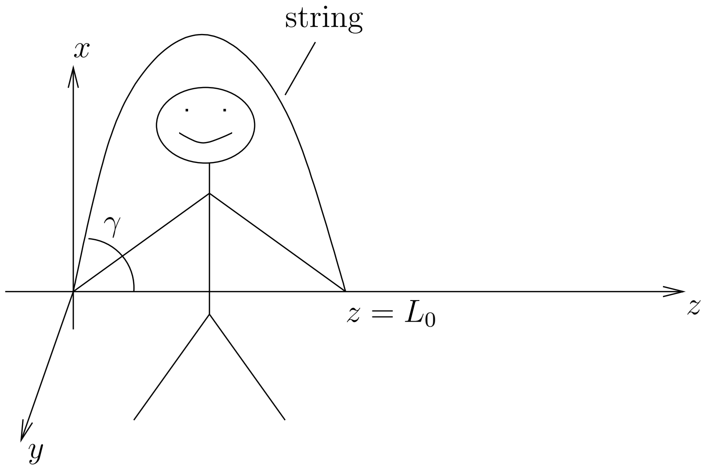

Figure 1: Kasey's relativistic rope

(a) Use (1) and the boundary condition at $\sigma = \sigma_1$ to find a condition on $\vec{F}(u)$.

(b) Write down the constraint on $\vec{F}(u)$ that arises from the parametrisation conditions

$$\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c} \frac{\partial \vec{X}}{\partial t} \right) = 1.$$

As an application, consider Kasey's attempts to use a relativistic open string as a jumping rope. For this purpose, she holds the open string (in three spatial dimensions) with her right hand at the origin $\vec{x}_1 = (0, 0, 0)$ and with her left hand at the point $z = L_0$ on the z axis, or $\vec{x}_2 = (0, 0, L_0)$ (Figure 1). As she starts jumping we observe that the tangent vector $\vec{X}'$ to the string at the origin rotates around the z axis forming an angle γ with it.

(c) Use the above information to write an expression for $\vec{F}'(u)$.

(d) Find σ_1 in terms of the length L_0 and the angle γ.

(e) Calculate $\vec{X}(t, \sigma)$ for the motion of Kasey's relativistic jumping rope.

(f) How is the energy distributed in the string as a function of z?