General relativity, exercise sheet 7.

HS 08 *Due:* Fri, November 14, 2008

1. Charged dust

Consider a charged dust consisting of particles of mass m and electric charge e.

i) Derive the equations of motion for $\rho(x)$ (mass density in the rest frame) and $u^\mu(x)$ (4-velocity) in an electromagnetic field $F_{\mu\nu}(x)$? Show that the 4-current $j^\mu(x)$ satisfies

$$j^\mu{}_{;\mu} = 0\,.$$

Hint: cf. (4.11)

ii) Let $T^{\mu\nu}_{\rm em}$, $T^{\mu\nu}_{\rm d}$ be the energy-momentum tensors of the electromagnetic field, resp. of the charged dust. Show that

$$(T^{\mu\nu}_{\rm em} + T^{\mu\nu}_{\rm d})_{;\nu} = 0\,.$$

Hint: In special relativity, $T^{\mu\nu}_{{\rm em},\nu} = -\frac{1}{c}F^{\mu\nu}j_\nu$.

2. Bound on the cosmological constant

i) How is the Poisson equation (5.12) modified by the introduction of the cosmological constant Λ in the Einstein field equations, see (5.14)?

ii) Show that the solution $\varphi = -G_0M/r$ for the gravitational potential generated by a point mass M is modified to

$$\varphi(\vec{x}) = -\frac{G_0M}{r} - \frac{1}{6}\Lambda c^2r^2\,.$$

iii) How small has Λ to be, so that its influence on the dynamics of the solar system is negligible?
Hint: Orbital radius of Pluto $r \cong 6\cdot10^{12}\,\mathrm{m}$, mass of the Sun $M \cong 2\cdot10^{30}\,\mathrm{kg}$, $G_0 \cong 6.7\cdot10^{-11}\,\mathrm{m^3\,kg^{-1}s^{-2}}$, $c \cong 3\cdot10^8\,\mathrm{m\,s^{-1}}$.