Institute for Theoretical Physics, ETH Zürich HS 2008

Exercises for "Phenomenology of Particle Physics I"

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Exercise 22

Starting from

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\sum_{s=\pm\frac{1}{2}}\left(e^{ipx}b_s^\dagger(\vec{p})v_s(p) + e^{-ipx}a_s(\vec{p})u_s(p)\right)$$

$$\overline{\psi(x)} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\sum_{s=\pm\frac{1}{2}}\left(e^{-ipx}b_s(\vec{p})\overline{v_s(p)} + e^{ipx}a_s^\dagger(\vec{p})\overline{u_s(p)}\right)$$

with $p^0 = E_{\vec{p}}$,$\overline{\psi} = \psi^\dagger\gamma^0$, $(\not{p} - m)u(p) = 0$, $(\not{p} + m)v(p) = 0$, show that if you postulate the anticommutation relations

$$\left\{a_r(\vec{p}), a_s^\dagger(\vec{q})\right\} = \left\{b_r(\vec{p}), b_s^\dagger(\vec{q})\right\} = \delta_{rs}(2\pi)^3 2E_{\vec{p}}\delta^3(\vec{p}-\vec{q})$$
$$\{a_r(\vec{p}), a_s(\vec{q})\} = \{b_r(\vec{p}), b_s(\vec{q})\} = \left\{a_r^\dagger(\vec{p}), a_s^\dagger(\vec{q})\right\} = \left\{b_r^\dagger(\vec{p}), b_s^\dagger(\vec{q})\right\} = \left\{a_r^{(\dagger)}(\vec{p}), b_s^{(\dagger)}(\vec{q})\right\} = 0$$

for the annihilation ($a(\vec{p})$, $b(\vec{p})$) and creation ($a^\dagger(\vec{p})$, $b^\dagger(\vec{p})$) operators of the particles (a) and of the antiparticles (b) you arrive at the equal time commutation relation

$$\left\{\psi(\vec{x},t), \overline{\psi(\vec{y},t)}\right\} = \gamma^0\delta^3(\vec{x}-\vec{y})$$

with all the other anticommutators equal to zero (make use of the completeness relations $\sum_s u_s(p)\overline{u_s(p)} = \not{p} + m$ and $\sum_s v_s(p)\overline{v_s(p)} = \not{p} - m$).

– please turn over –

Exercise 23

Demonstrate that the anticommutation relations of the Dirac field ψ shown in the previous exercise guarantee the microcausality of physical observables of the Dirac theory, i.e

$$\left[\overline{\psi(\vec{x},t)}\Gamma_1\psi(\vec{x},t),\overline{\psi(\vec{y},t)}\Gamma_2\psi(\vec{y},t)\right]=0 \qquad \text{for} \quad \vec{x}\neq\vec{y}$$

where $\Gamma_1,\Gamma_2\in\{1,\gamma^\mu,\gamma^5,\gamma^\mu\gamma^5,i/2[\gamma^\mu,\gamma^\nu]\}$. Use the decomposition of the commutator as in $[AB,CD]=A\{B,C\}D-AC\{B,D\}-C\{A,D\}B+\{C,A\}DB$.