Quantum Field Theory I, Exercise Set 6.

HS 08 *Due:* 6/7 November 2008

1. Quantization of the Massive Vector Field

(i) Find the Euler-Lagrange equations for a massive vector field W^μ, $\mu = 0, \ldots, 3$, with Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} W_\mu W^\mu \,,$$

where $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ and $m > 0$. Show that they are equivalent to

$$\partial_\mu W^\mu = 0 \,, \tag{1}$$

$$(\Box + m^2) W^\mu = 0 \,. \tag{2}$$

(ii) Convience yourself that this Lagrangian is not gauge invariant. Discuss whether a Hamiltonian formulation is possible.

(iii) A Lagrangian density yielding (2), but not the condition (1), is

$$\mathcal{L} = -\frac{1}{2} (\partial_\mu W_\nu)(\partial^\mu W^\nu) + \frac{m^2}{2} W_\mu W^\mu \,.$$

Describe how the Gupta-Bleuler method can be used to quantize W^μ. Use an expansion similar to (6.7) in the lecture notes with polarizations as in (6.8)-(6.9) with

$$\varepsilon_0(k) := \frac{k}{m} \,, \quad k \in V_m \,.$$

Imposing the condition (1), i.e., $\partial_\mu W^{+\mu}(x) |\psi\rangle = 0$, conclude that the inner product is positive definite.

(iv) Compute the Feynman propagator.