Statistical Physics
Exercise Sheet 1

Exercise 1.1 Thermodynamics of a magnetic system

a) Consider a long empty inductor of length l, cross-section surface F and number of turns N with a current I flowing. We now fill the solenoid uniformly with magnetic material. Show that the work done by the battery in the infinitesimal time interval dt is $\delta W_B = \vec{H} \cdot d\vec{\mathcal{M}}$ where $\vec{\mathcal{M}} = \Omega \vec{M}$ and $\Omega = Fl$ is the volume of the magnetic material. Consequently, we have $dU_B = \delta Q + \vec{H} \cdot d\vec{\mathcal{M}}$.

Hint: Use Ampere's and Faraday's law.

b) Consider a fixed magnetic field $\vec{H}$. Show that the work done by an external mechanical agency when a magnetic dipole $\vec{\mathcal{M}}$ is displaced by $d\vec{l}$ in the external magnetic field is $\delta W_A = -\vec{\mathcal{M}} \cdot d\vec{H}$. Consequently we have $dU_A = \delta Q - \vec{\mathcal{M}} \cdot d\vec{H}$.

c) Consider a magnetic system as described in a) or b). Show that the following Maxwell relations hold:

$$\left(\frac{\partial T}{\partial \mathcal{M}}\right)_S = \left(\frac{\partial H}{\partial S}\right)_{\mathcal{M}} , \tag{1}$$

and

$$\left(\frac{\partial \mathcal{M}}{\partial T}\right)_H = \left(\frac{\partial S}{\partial H}\right)_T . \tag{2}$$

Exercise 1.2 Ideal paramagnet

In this exercise we study the thermodynamics of an ideal classical paramagnet of unit volume specified by the thermal and the caloric equation of state:

$$M(T,H) = Nm\left[\coth\left(\frac{mH}{k_BT}\right) - \frac{k_BT}{mH}\right] , \tag{3}$$

$$U(T,H) = C_M T . \tag{4}$$

In the notation of the previous exercise $U = U_A$ and $dU = \delta Q + HdM$.

a) Find the curves of the reversible adiabatics and isotherms in the M-H and in the M-T diagram for the cases (i) $mH \gg k_BT$ and (ii) $mH \ll k_BT$.

Hint: Use $\coth(x) = \frac{1}{x} + \frac{x}{3} + \cdots$.

b) Construct a Carnot engine using the ideal paramagnet as an operating material between two reservoirs 1 and 2 of temperature T_1 and T_2, respectively ($T_1 > T_2$). Calculate the efficiency of the engine for the two cases (i) and (ii) in a).

c) Calculate the entropy $S(U,M)$ for the two cases (i) and (ii) in a).