Statistical Physics
Exercise Sheet 6

Exercise 6.1 Mixture of Two Ideal Gases in a Harmonic Trap

Consider a mixture of two *different* ideal gases A and B in a harmonic trap. The Hamiltonian is given by

$$\mathcal{H}(p,q) = \sum_{i=1}^{N_A} \left\{ \frac{\mathbf{p}_{A,i}^2}{2m_A} + \frac{D_A}{2}\mathbf{q}_{A,i}^2 \right\} + \sum_{i=1}^{N_B} \left\{ \frac{\mathbf{p}_{B,i}^2}{2m_B} + \frac{D_B}{2}\mathbf{q}_{B,i}^2 \right\}. \tag{1}$$

The system is considered to be isolated; i.e., the microcanonical ensemble is to be used in the following.

a) Calculate the entropy of the system.

b) Find the equilibrium value of $E_A = \langle \mathcal{H}_A \rangle$, the energy of the gas A in the mixture.

c) Find the spatial density distribution $n(\mathbf{x})$.

Hint: Use Stirling's formula for the Gamma function,

$$\Gamma(z) \sim (2\pi)^{1/2} e^{-z} z^{z-1/2}, \; z \gg 1. \tag{2}$$

Exercise 6.2 Relativistic Ideal Gas

Calculate $\langle E \rangle$ for a relativistic ideal gas and analyze the low and high temperature limits. *Hint*: Use the equipartition law.

Exercise 6.3 The Ising Paramagnet

Consider N localized magnetic moments which can assume the values $s_i = \pm s$. In the presence of a magnetic field h the Hamiltonian is given by

$$\mathcal{H} = -\sum_i h s_i. \tag{3}$$

Calculate the free energy $F(T,h)$, the caloric and thermal equations of state, the specific heat $C(T,h)$ and the magnetic susceptibility $\chi(T,h)$.