Algorithms and Data Structures in C++

Complexity analysis

- Answers the question “How does the time needed for an algorithm scale with the problem size *N*?”
  - Worst case analysis: maximum time needed over all possible inputs
  - Best case analysis: minimum time needed
  - Average case analysis: average time needed
  - Amortized analysis: average over a sequence of operations
- Usually only worst-case information is given since average case is much harder to estimate.

The O notation

- Is used for worst case analysis:

  An algorithm is $O(f(N))$ if there are constants c and N_0, such that for $N \geq N_0$ the time to perform the algorithm for an input size N is bounded by $t(N) < c\, f(N)$

- Consequences
  - $O(f(N))$ is identically the same as $O(a\, f(N))$
  - $O(a\, N^x + b\, N^y)$ is identically the same as $O(N^{\max(x,y)})$
  - $O(N^x)$ implies $O(N^y)$ for all $y \geq x$

The Ω and Θ notations

- Ω is used for best case analysis:

  An algorithm is $\Omega(f(N))$ if there are constants c and N_0, such that for $N \geq N_0$ the time to perform the algorithm for an input size N is bounded by $t(N) > c\, f(N)$

- Θ is used if worst and best case scale the same

  Data structures and algorithms in the C++ standard library An algorithm is $\Theta(f(N))$ if it is $\Omega(f(N))$ and $O(f(N))$

Time assuming 1 billion operations per second

Complexity	N=10	10^2	10^3	10^4	10^5	10^6
1	1 ns	1 ns	1 ns	1 ns	1 ns	1ns
ln N	3 ns	7 ns	10 ns	13 ns	17 ns	20 ns
N	10 ns	100 ns	1 μs	10 μs	100 μs	1 ms
N log N	33 ns	664 ns	10 μs	133 μs	1.7 ms	20 ms
N^2	100 ns	10 μs	1 ms	100 ms	10 s	17 min
N^3	1 μs	1 ms	1 s	17 min	11.5 d	31 a
2^N	1 μs	10^{14} a	10^{285} a	10^{2996} a	10^{30086} a	10^{301013} a

Which algorithm do you prefer?

- When do you pick algorithm A, when algorithm B? The complexities are listed below

Algorithm A	Algorithm B	Which do you pick?
O(ln N)	O(N)	
O(ln N)	N	
O(ln N)	1000 N	
ln N	O(N)	
1000 ln N	O(N)	
ln N	N	
ln N	1000 N	
1000 ln N	N	

Complexity: example 1

- What is the O, Ω and Θ complexity of the following code?

```
double x;
std::cin >> x;
std::cout << std::sqrt(x);
```

Complexity: example 2

- What is the O, Ω and Θ complexity of the following code?

```
unsigned int n;
std::cin >> n;
for (int i=0; i<n; ++i)
  std::cout << i*i << "\n";
```

Complexity: example 3

- What is the O, Ω and Θ complexity of the following code?

```
unsigned int n;
std::cin >> n;
for (int i=0; i<n; ++i) {
  unsigned int sum=0;
  for (int j=0; j<i; ++j)
    sum += j;
  std::cout << sum << "\n";
}
```

Complexity: example 4

- What is the O, Ω and Θ complexity of the following two segments?
  - Part 1:

```
unsigned int n;
std::cin >> n;
double* x=new double[n]; // allocate array of n numbers
for (int i=0; i<n; ++i)
  std::cin >> x[i];
```

  - Part 2:

```
double y;
std::cin >> y;
for (int i=0; i<n; ++i)
  if (x[i]==y) {
    std::cout << i << "\n";
    break;
  }
```

Complexity: adding to an array (simple way)

- What is the complexity of adding an element to the end of an array?
  - allocate a new array with N+1 entries
  - copy N old entries
  - delete old arrray
  - write (N+1)-st element
- The complexity is O(N)

Complexity: adding to an array (clever way)

- What is the complexity of adding an element to the end of an array?
  - allocate a new array with 2N entries, but mark only N+1 as used
  - copy N old entries
  - delete old arrray
  - write (N+1)-st element
- The complexity is O(N), but let's look at the next elements added:
  - mark one more element as used
  - write additional element
- The complexity here is O(1)
- The amortized (averaged) complexity for N elements added is

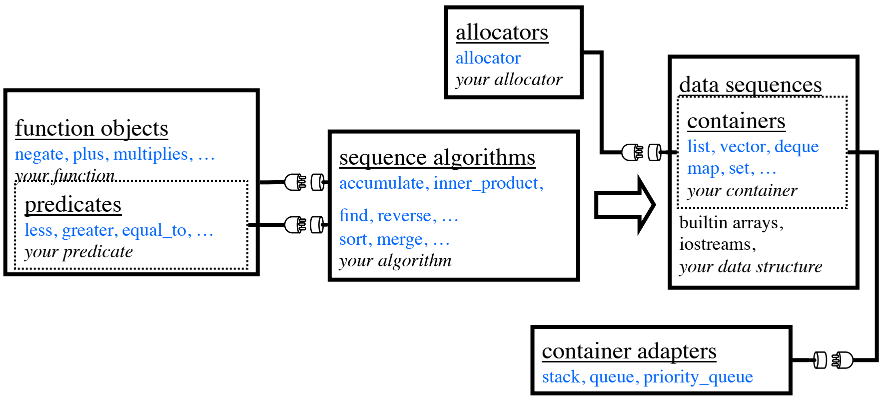
$$\frac{1}{N}\left(O(N)+(N-1)O(1)\right)=O(1)$$

STL: Standard Template Library

- Most notable example of generic programming
- Widely used in practice
- Theory: Stepanov, Musser; Implementation: Stepanov, Lee

- *Standard* Template Library
  - Proposed to the ANSI/ISO C++ Standards Committee in 1994.
  - After small revisions, part of the official C++ standard in 1997.

The standard C++ library



The `string` and `wstring` classes

- are very useful class to manipulate strings
  - `string` for standard ASCII strings (e.g. “English”)
  - `wstring` for wide character strings (e.g. “日本語”)
- Contains many useful functions for string manipulation
  - Adding strings
  - Counting and searching of characters
  - Finding substrings
  - Erasing substrings
  - …
- Since this is not very important for numerical simulations I will not go into details. Please read your C++ book

The `pair` template

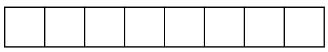
-
```
template <class T1, class T2> class pair {
public:
  T1 first;
  T2 second;
  pair(const T1& f, const T2& s)
   : first(f), second(s)
  {}
};
```
- will be useful in a number of places

Data structures in C++

- We will discuss a number of data structures and their implementation in C++:
- Arrays:
  - C array
  - `vector`
  - `valarray`
  - `deque`
- Linked lists:
  - `list`
- Trees
  - `map`
  - `set`
  - `multimap`
  - `multiset`
- Queues and stacks
  - `queue`
  - `priority_queue`
  - `stack`

The array or vector data structure

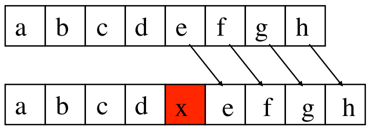
- An array/vector is a consecutive range in memory



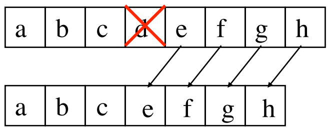
- Advantages
  - Fast $O(1)$ access to arbitrary elements: `a[i]` is `*(a+i)`
  - Profits from cache effects
  - Insertion or removal at the end is $O(1)$
  - Searching in a sorted array is $O(\ln N)$
- Disadvantage
  - Insertion and removal at arbitrary positions is $O(N)$

Slow O(*N*) insertion and removal in an array

- Inserting an element
  - Need to copy O(N) elements

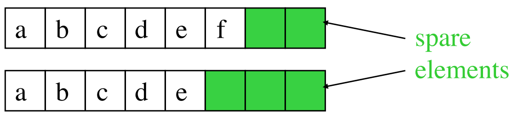


- Removing an element
  - Also need to copy O(N) elements

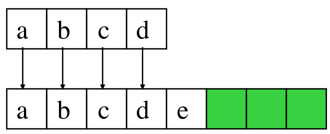


Fast O(1) removal and insertion at the end of an array

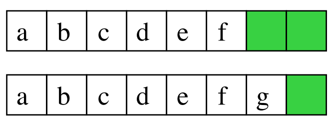
- Removing the last element
  - Just change the size
    - Capacity 8, size 6:
    - Capacity 8, size 5:



- Inserting elements at the end
  - Is amortized O(1)
    - first double the size and copy in O(N):



    - then just change the size:

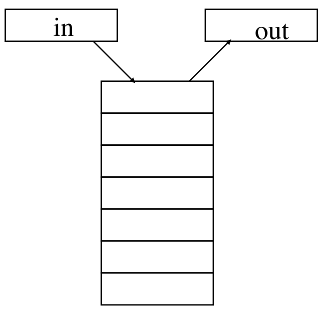


The deque data structure (double ended queue)

- Is a variant of an array, more complicated to implement
  - See a data structures book for details
- In addition to the array operations also the insertion and removal at beginning is O(1)
- Is needed to implement queues

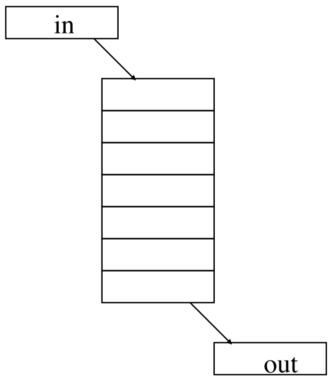
The stack data structure

- Is like a pile of books
  - LIFO (last in first out): the last one in is the first one out
- Allows in O(1)
  - Pushing an element to the top of the stack
  - Accessing the top-most element
  - Removing the top-most element



The queue data structure

- Is like a queue in the Mensa
  - FIFO (first in first out): the first one in is the first one out
- Allows in $O(1)$
  - Pushing an element to the end of the queue
  - Accessing the first and last element
  - Removing the first element

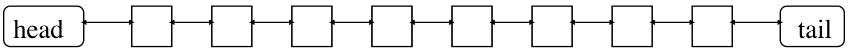


The priority queue data structure

- Is like a queue in the Mensa, but professors are allowed to go to the head of the queue (not passing other professors though)
  - The element with highest priority (as given by the < relation) is the first one out
  - If there are elements with equal priority, the first one in the queue is the first one out
- There are a number of possible implementations, look at a data structure book for details

The linked list data structure

- An linked list is a collection of objects linked by pointers into a one-dimensional sequence



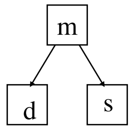
- Advantages
  - Fast $O(1)$ insertion and removal anywhere
    - Just reconnect the pointers
- Disadvantage
  - Does not profit from cache effects
  - Access to an arbitrary element is $O(N)$
  - Searching in a list is $O(N)$

The tree data structures

- An array needs
  - $O(N)$ operations for arbitrary insertions and removals
  - $O(1)$ operations for random access
  - $O(N)$ operations for searches
  - $O(\ln N)$ operations for searches in a sorted array
- A list needs
  - $O(1)$ operations for arbitrary insertions and removals
  - $O(N)$ operations for random access and searches
- What if both need to be fast? Use a tree data structure:
  - $O(\ln N)$ operations for arbitrary insertions and removals
  - $O(\ln N)$ operations for random access and searches

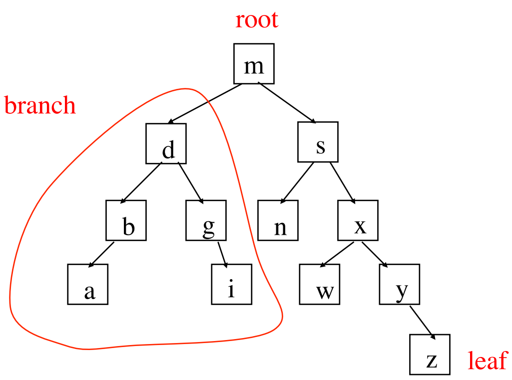
A node in a binary tree

- Each node is always linked to two child nodes
  - The left child is always smaller
  - The right child node is always larger



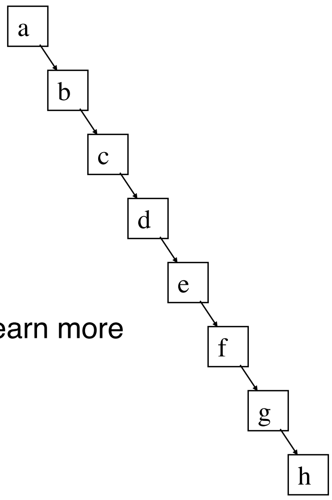
A binary tree

- Can store $N=2^{n-1}$ nodes in a tree of height n
  - Any access needs at most $n = O(\ln N)$ steps
- Example: a tree of height 5 with 12 nodes



Unbalanced trees

- Trees can become unbalanced
  - Height is no longer O(ln N) but O(N)
  - All operations become O(N)
- Solutions
  - Rebalance the tree
  - Use self-balancing trees
- Look into a data structures book to learn more



Tree data structures in the C++ standard

- Fortunately the C++ standard contains a number of self-balancing tree data structures suitable for most purposes:
  - `set`
  - `multiset`
  - `map`
  - `multimap`
- But be aware that computer scientists know a large number of other types of trees and data structures
  - Read the books
  - Ask the experts

The container concept in the C++ standard

- Containers are sequences of data, in any of the data structures
  - `vector<T>` is an array of elements of type T
  - `list<T>` is a doubly linked list of elements of type T
  - `set<T>` is a tree of elements of type T

  …
- The standard assumes the following requirements for the element T of a container:
  - default constructor `T()`
  - assignment `T& operator=(const T&)`
  - copy constructor `T(const T&)`
  - Note once again that assignment and copy have to produce identical copy: in the Penna model the copy constructor should not mutate!

Connecting Algorithms to Sequences

```
find( s, x ) :=
    pos ← start of s
    while pos not at end of s
        if element at pos in s == x
            return pos
        pos ← next position
    return pos
```

```
int find( char const(&s)[4], char x )
{
    int pos = 0;
    while (pos != sizeof(s))
    {
        if ( s[pos] == x )
            return pos;
        ++pos;
    }
    return pos;
}
```

```
struct node
{
    char value;
    node* next;
};
```

```
node* find( node* const s, char x )
{
    node* pos = s;
    while (pos != 0)
    {
        if ( pos->value == x )
            return pos;
        pos = pos->next;
    }
    return pos;
}
```

Connecting Algorithms to Sequences

```
find( s, x ) :=
    pos ← start of s
    while pos not at end of s
          if element at pos in s == x
                    return pos
          pos ← next position
    return pos
```

```
char* find(char const(&s)[4], char x)
{
    char* pos = s;
    while (pos != s + sizeof(s))
    {
       if ( *pos == x )
          return pos;
       ++pos;
    }
    return pos;
}
```

```
struct node
{
    char value;
    node* next;
};
```

```
node* find( node* const s, char x )
{
    node* pos = s;
    while (pos != 0)
    {
         if ( pos->value == x )
                   return pos;
         pos = pos->next;
    }
    return pos;
}
```

Connecting Algorithms to Sequences

```
find( s, x ) :=
    pos ← start of s
    while pos not at end of s
          if element at pos in s == x
                    return pos
          pos ← next position
    return pos
```

```
char* find(char const(&s)[4], char x)
{
    char* pos = s;
    while (pos != s + sizeof(s))
    {
       if ( *pos == x )
          return pos;
       ++pos;
    }
    return pos;
}
```

```
struct node
{
    char value;
    node* next;
};
```

```
node* find( node* const s, char x )
{
    node* pos = s;
    while (pos != 0)
    {
         if ( pos->value == x )
                   return pos;
         pos = pos->next;
    }
    return pos;
}
```

NxM Algorithm Implementations?

1. find
2. copy
3. merge
4. transform

⋮

N. accumulate

1. vector
2. list
3. deque
4. set
5. map
6. char[5]

⋮

M. foobar

F. T. S. E.

Fundamental Theorem of Software Engineering

"We can solve any problem by introducing an extra level of indirection"

--Butler Lampson

Andrew Koenig

Iterators to the Rescue

- Define a common interface for
  - traversal
  - access
  - positional comparison
- Containers provide iterators
- Algorithms operate on pairs of iterators

```
template <class Iter, class T>
Iter find( Iter start, Iter finish, T x )
{
   Iter pos = start;
   for (; pos != end; ++pos)
   {
      if ( *pos == x )
         return pos;
   }
   return pos;
}
```

```
struct node_iterator
{
   // ...
   char& operator*() const
   { return n->value; }

   node_iterator& operator++()
   { n = n->next; return *this; }
private:
   node* n;
};
```

Describe Concepts for std::find

```
template <class Iter, class T>
Iter find(Iter start, Iter finish, T x)
{
   Iter pos = start;
   for (; pos != finish; ++pos)
   {
      if ( *pos == x )
         return pos;
   }
   return pos;
}
```

- Concept Name?
- Valid expressions?
- Preconditions?
- Postconditions?
- Complexity guarantees?
- Associated types?

Traversing an array and a linked list

- Two ways for traversing an array
  - Using an index:

```
T* a = new T[size];
for (int n=0;n<size;++n)
  cout << a[n];
```

  - Using pointers:

```
for (T* p = a;
     p !=a+size;
     ++p)
  cout << *p;
```

- Traversing a linked list

```
template <class T> struct node
{
  T value;  // the element
  node<T>* next;  // the next Node
};

template<class T> struct list
{
  node<T>* first;
};
list<T> l;
…
for (mode<T>* p=l.first;
       p!=0;
       p=p->next)
    cout << p->value;
```

Generic traversal

- Can we traverse a vector and a list in the same way?
- Instead of

```
for (T* p = a;
     p !=a+size;
     ++p)
  cout << *p;
```

- We want to write

```
for (iterator p = a.begin();
     p !=a.end();
     ++p)
  cout << *p;
```

- Instead of

```
for (node<T>* p=l.first;
     p!=0;
     p=p->next)
  cout << p->value;
```

- We want to write

```
for (iterator p = l.begin();
     p !=l.end();
     ++p)
  cout << *p;
```

Implementing iterators for the array

```
template<class T>
  class Array {
  public:
    typedef T* iterator;
    typedef unsigned size_type;
    Array();
    Array(size_type);

    iterator begin()
    { return p_;}
    iterator end()
    { return p_+sz_;}

  private:
    T* p_;
    size_type sz_;
  };
```

- Now allows the desired syntax:

```
for (Array<T>::iterator p =
  a.begin();
     p !=a.end();
     ++p)
  cout << *p;
```

- Instead of

```
for (T* p = a.p_;
     p !=a.p_+a.sz_;
     ++p)
  cout << *p;
```

Implementing iterators for the linked list

```
template <class T>
  struct node_iterator {
  Node<T>* p;
  node_iterator(Node<T>* q)
   : p(q) {}

  node_iterator<T>& operator++()
  { p=p->next;}

  T* operator ->()
  { return &(p->value);}

  T& operator*()
  { return p->value;}

  bool operator!=(const
        node_iterator<T>& x)
  { return p!=x.p;}

  // more operators missing …
  };
```

```
template<class T>
  class list {
   Node<T>* first;
  public:
    typedef node_iterator<T> iterator;

   iterator begin()
    { return iterator(first);}

    iterator end()
    { return iterator(0);}
  };
```

- Now also allows the desired syntax:

```
for (List<T>::iterator p = l.begin();
     p !=l.end();
      ++p)
  cout << *p;
```

Iterators

- have the same functionality as pointers
- including pointer arithmetic!
  - `iterator a,b; cout << b-a;` // # of elements in [a,b[
- exist in several versions
  - forward iterators … move forward through sequence
  - backward iterators … move backwards through sequence
  - bidirectional iterators … can move any direction
  - input iterators … can be read: `x=*p;`
  - output iterators … can be written: `*p=x;`
- and all these in const versions (except output iterators)

Container requirements

- There are a number of requirements on a container that we will now discuss based on the handouts

Containers and sequences

- A container is a collection of elements in a data structure
- A sequence is a container with a linear ordering (not a tree)
  - vector
  - deque
  - list
- An associative container is based on a tree, finds element by a key
  - map
  - multimap
  - set
  - multiset
- The properties are defined on the handouts from the standard
  - A few special points mentioned on the slides

Sequence constructors

- A sequence is a linear container (vector, deque, list,…)
- Constructors
  - `container()` … empty container
  - `container(n)` … n elements with default value
  - `container(n,x)` … n elements with value x
  - `container(c)` … copy of container c
  - `container(first,last)` … first and last are iterators
    - container with elements from the range [first,last[
- Example:

```
std::list<double> l;
// fill the list
…
// copy list to a vector
std::vector<double> v(l.begin(),l.end());
```

Direct element access in deque and vector

- Optional element access (not implemented for all containers)
  - `T& container[k]` … k-th element, no range check
  - `T& container.at(k)` … k-th element, with range check
  - `T& container.front()` … first element
  - `T& container.back()` … last element

Inserting and removing at the beginning and end

- For all sequences: inserting/removing at end
  - `container.push_back(T x)` // add another element at end
  - `container.pop_back()` // remove last element

- For list and deque (stack, queue)
  - `container.push_first(T x)` // insert element at start
  - `container.pop_first()` // remove first element

Inserting and erasing anywhere in a sequence

- List operations (slow for vectors, deque etc.!)
  - `insert (p,x)` // insert x before p
  - `insert(p,n,x)` // insert n copies of x before p
  - `insert(p,first,last)` // insert [first,last[before p
  - `erase(p)` // erase element at p
  - `erase(first,last)` // erase range[first,last[
  - `clear()` // erase all

Vector specific operations

- Changing the size
  - `void resize(size_type)`
  - `void reserve(size_type)`
  - `size_type capacity()`
- Note:
  - `reserve` and `capacity` regard memory `allocated` for vector!
  - `resize` and `size` regard memory currently used for vector data
- Assignments
  - `container = c` … copy of container c
  - `container.assign(n)` …assign n elements the default value
  - `container.assign(n,x)` … assign n elements the value x
  - `container.assign(first,last)` … assign values from the range [first,last[
- Watch out: assignment does not allocate, do a resize before!

The `valarray` template

- acts like a vector but with additional (mis)features:
  - No iterators
  - No reserve
  - Resize is fast but erases contents
- for numeric operations are defined:

```
std::valarray<double> x(100), y(100), z(100);
x=y+exp(z);
```

  - Be careful: it is not the fastest library!
  - We will learn about faster libraries later

Sequence adapters: `queue` and `stack`

- are based on deques, but can also use vectors and lists
  - `stack` is first in-last out
  - `queue` is first in-first out
  - `priority_queue` prioritizes with < operator
- stack functions
  - `void push(const T& x)` … insert at top
  - `void pop()` … removes top
  - `T& top()`
  - `const T& top() const`
- queue functions
  - `void push(const T& x)` … inserts at end
  - `void pop()` … removes front
  - `T& front()`, `T& back()`, `const T& front()`, `const T& back()`

`list` -specific functions

- The following functions exist only for std::list:
  - `splice`
    - joins lists without copying, moves elements from one to end of the other
  - `sort`
    - optimized sort, just relinks the list without copying elements
  - `merge`
    - preserves order when "splicing" sorted lists
  - `remove(T x)`
  - `remove_if(criterion)`
    - criterion is a function object or function, returning a bool and taking a `const T&` as argument, see Penna model
    - example:

```
bool is_negative(const T& x) { return x<0;}
```

    - can be used like

```
list.remove_if(is_negative);
```

The `map` class

- implements associative arrays
  - ```
    map<std::string,long> phone_book;
    phone_book["Troyer"] = 32589;
    phone_book["Heeb"] = 32591;
    if(phone_book[name])
      cout << "The phone number of " << name << " is "
           << phone_book[name];
    else
      cout << name << "\'s phone number is unknown!';
    ```
- is implemented as a tree of `pair`s
- Take care:
  - `map<T1,T2>::value_type` is `pair<T1,T2>`
  - `map<T1,T2>::key_type` is `T1`
  - `map<T1,T2>::mapped_type` is `T2`
  - `insert`, `remove`, … are sometimes at first sight confusing for a map!
    ```

Other tree-like containers

- `multimap`
  - can contain more than one entry (e.g. phone number) per key
- `set`
  - unordered container, each entry occurs only once
- `multiset`
  - unordered container, multiple entries possible
- extensions are no problem
  - if a data structure is missing, just write your own
  - good exercise for understanding of containers

Search operations in trees

- In a map<K,V>, K is the key type and V the mapped type
  - Attention: iterators point to pairs
- In a map<T>, T is the key type and also the value_type
- Fast O(log *N*) searches are possible in trees:
  - `a.find(k)` returns an iterator pointing to an element with key k or end() if it is not found.
  - `a.count(k)` returns the number of elements with key k.
  - `a.lower_bound(k)` returns an iterator pointing to the first element with key >= k.
  - `a.upper_bound(k)` returns an iterator pointing to the first element with key > k.
  - `a.equal_range(k)` is equivalent to but faster than `std::make_pair(a.lower_bound(k) , a.upper_bound(k))`

Search example in a tree

- Look for all my phone numbers:
  - ```
    // some typedefs
    typedef multimap<std::string, int> phonebook_t;
    typedef phonebook_t::const_iterator IT;
    typedef phonebook_t::value_type value_type;

    // the phonebook
    phonebook_t phonebook;

    // fill the phonebook
    phonebook.insert(value_type("Troyer",32589));
    …

    // search all my phone numbers
    pair< IT,IT> range =  phonebook.equal_range("Troyer");

    // print all my phone numbers
    for (IT it=range.first; it != range.second;++it)
      cout << it->second << "\n";
    ```

Almost Containers

- C-style array
- `string`
- `valarray`
- `bitset`

- They all provide almost all the functionality of a container
- They can be used like a container in many instances, but not all
  - ```
    int x[5] = {3,7,2,9,4};
    vector<int> v(x,x+5);
    ```
  - uses `vector(first,last)`, pointers are also iterators!

The generic algorithms

- Implement a big number of useful algorithms
- Can be used on any container
  - rely only on existence of iterators
  - “container-free algorithms”
  - now all the fuss about containers pays off!
- Very useful
- Are an excellent example in generic programming
- We will use them now for the Penna model
  That’s why we did not ask you to code the Population class for the Penna model yet!

Example: `find`

- A generic function to find an element in a container:
  -
```
list<string> fruits;
list<string>::const_iterator found =
  find(fruits.begin(),fruits.end(),"apple");
if (found==fruits.end()) // end means invalid iterator
 cout << "No apple in the list";
else
  cout << "Found it: " << *found << "\n";
```
- find declared and implemented as
  -
```
template <class In, class T>
  In find(In first, In last, T v) {
    while (first != last  && *first != v)
      ++first;
    return first;
  }
```

Example: `find_if`

- takes predicate (function object or function)
  - ```
    bool favorite_fruits(const std::string& name)
    { return (name=="apple" || name == "orange");}
    ```
- can be used with `find_if` function:
  - ```
    list<string>::const_iterator found =
      find_if(fruits.begin(),fruits.end(),favorite_fruits);
    if (found==fruits.end())
      cout << "No favorite fruits in the list";
    else
      cout << "Found it: " << *found << "\n";
    ```
- find_if declared and implemented as as
  - ```
    template <class In, class Pred>
      In find_if(In first, In last, Pred p) {
        while (first != last && !p(*first) )
          ++first;
        return first;
    }
    ```

Member functions as predicates

- We want to find the first pregnant animal:
  - ```
    list<Animal> pop;
    find_if(pop.begin(),pop.end(),is_pregnant)
    ```
- This does not work as expected, it expects
  - `bool is_pregnant(const Animal&);`
- We want to use
  - `bool Animal::pregnant() const`
- Solution: mem_fun_ref function adapter
  - ```
    find_if(pop.begin(),pop.end(),
                     mem_fun_ref(&Animal::pregnant));
    ```
- Many other useful adapters available
  - Once again: please read the books before coding your own!
    ```

push_back and back_inserter

- Attention:
  - ```
    vector<int> v,w;
    for (int k=0;k<100;++k){
      v[k]=k; //error: v is size 0!
      w.push_back(k); // OK:grows the array and assigns
    }
    ```
- Same problem with copy:
  - ```
    vector<int> v(100), w(0);
    copy(v.begin(),v.end(),w.begin()); // problem: w of size 0!
    ```
- Solution1: vectors only
  - w.resize(v.size()); copy(v.begin(),v.end(),w.begin());
- Solution 2: elegant
  - copy(v.begin(),v.end(),back_inserter(w)); // uses push_back
- also push_front and front_inserter for some containers

Penna Population

- easiest modeled as
  - `class Population : public list<Animal> {…}`
- Removing dead:
  - `remove_if(mem_fun_ref(&Animal::is_dead));`
- Removing dead, and others with probability N/N0:
  - `remove_if(animal_dies(N/N0));`
  - where `animal_dies` is a function object taking N/N0 as parameter
- Inserting children:
  - cannot go into same container, as that might invalidate iterators:
    ```
    vector<Animal> children;
    for(const_iterator a=begin();a!=end();++a)
      if(a->pregnant())
        children.push_back(a->child());
    copy(children.begin(),children.end(),
        back_inserter(*this);
    ```

The binary search

- Searching using binary search in a sorted vector is $O(\ln N)$
- Binary search is recursive search in range [begin,end[
  - If range is empty, return
  - Otherwise test middle=begin+(end-begin)/2
    - If the element in the middle is the search value, we are done
    - If it is larger, search in [begin,middle[
    - If it is smaller, search in [middle,end[
- The search range is halved in every step and we thus need at most $O(\ln N)$ steps

Example: lower_bound

```
template<class IT, class T>
IT lower_bound(IT first, IT last, const T& val) {
  typedef typename iterator_traits<IT>::difference_type dist_t;
  dist_t len = distance(first, last); // generic function for last-first
  dist_t half;
  IT middle;
  while (len > 0) {
    half = len >> 1; // faster version of half=len/2
    middle = first;
    advance(middle, half);// generic function for middle+=half
    if (*middle < val) {
      first = middle;
      ++first;
      len = len - half - 1;
    }
    else
      len = half;
  }
  return first;
}
```

Algorithms overview

- Nonmodifying
  - `for_each`
  - `find, find_if, find_first_of`
  - `adjacent_find`
  - `count, count_if`
  - `mismatch`
  - `equal`
  - `search`
  - `find_end`
  - `search_n`
- Modifying
  - `transform`
  - `copy, copy_backward`
  - `swap, iter_swap, swap_ranges`
  - `replace, replace_if, replace_copy, replace_copy_if`
  - `fill, fill_n`
  - `generate, generate_n`
  - `remove, remove_if, remove_copy, remove_copy_if`
  - `unique, unique_copy`
  - `reverse, reverse_copy`
  - `rotate, rotate_copy`
  - `random_shuffle`

Algorithms overview (continued)

- Sorted Sequences
  - `sort,stable_sort`
  - `partial_sort, partial_sort_copy`
  - `nth_element`
  - `lower_bound, upper_bound`
  - `equal_range`
  - `binary_search`
  - `merge, inplace_merge`
  - `partition, stable_partition`
- Permutations
  - `next_permutation`
  - `prev_permutation`
- Set Algorithms
  - `includes`
  - `set_union`
  - `set_intersection`
  - `set_difference`
  - `set_symmetric_difference`
- Minimum and Maximum
  - `min`
  - `max`
  - `min_element`
  - `max_element`
  - `lexicographical_compare`

Exercise

- Code the population class for the Penna model based on a standard container
- Use function objects to determine death

- In the example we used a loop.
  - Can you code the population class without using any loop?
  - This would increase the reliability as the structure is simpler!

- Also add fishing in two variants:
  - fish some percentage of the whole population
  - fish some percentage of adults only

- Read Penna's papers and simulate the Atlantic cod!
  Physica A, **215**, 298 (1995)

stream iterators and Shakespeare

- Iterators can also be used for streams and files
  - `istream_iterator`
  - `ostream_iterator`

- Now you should be able to understand Shakespeare:

```
int main()
 {
    vector<string> data;
    copy(istream_iterator<string>(cin),istream_iterator<string>(),
         back_inserter(data));
    sort(data.begin(), data.end());
    unique_copy(data.begin(),data.end(),ostream_iterator<string>(cout,"\n"));
 }
```

Summary

- Please read the sections on
  - containers
  - iterators
  - algorithms
- in Stroustrup or Lippman (3rd editions only!)

- Examples of excellent class and function designs
- Before writing your own functions and classes:
  Check the standard C++ library!
- When writing your own functions/classes:
  Try to emulate the design of the standard library
- Don't forget to include the required headers:
  - `<algorithm>`, `<functional>`, `<map>`, `<iterators>`, … as needed