

Exercise 6.1 The Ising Paramagnet

Consider N localized non-interacting magnetic moments which can assume the values $s_i = \pm s$. In the presence of a magnetic field h the Hamiltonian is given by

$$\mathcal{H} = - \sum_i h s_i.$$

Calculate the free energy $F(T, h)$, the caloric and thermal equations of state, the heat capacity $C(T, h)$ and the magnetic susceptibility $\chi(T, h)$.

Exercise 6.2 Non-interacting Particles in the Gravitational Field

Consider a gas of non-interacting particles at fixed temperature T in the gravitational field $V_{\text{grav}}(x, y, z \geq 0) = gz$ with constant $g > 0$, confined to a vertical, cylindrical vessel (radius R) of infinite height.

- Using the canonical ensemble, find the Helmholtz free energy, the entropy, the heat capacity, and the internal energy of this system.
- Interpret the result of the heat capacity via the equipartition law.
- Consider the system from the viewpoint of local thermal equilibrium. Determine the local (one-particle) density $n(z)$ and the local pressure $p(z)$ and show that the equation of states $p(z) = k_B T n(z)$ holds. Find the expression of the local internal energy density $u(z)$. The local version of the relation (1.17) in the lecture notes takes the form

$$c_p = \left(\frac{\partial u}{\partial T} \right)_n + \left\{ \left(\frac{\partial U}{\partial V} \right)_T + p \right\} \alpha = \left(\frac{\partial u}{\partial T} \right)_n + T \left(\frac{\partial p}{\partial T} \right)_n \alpha. \quad (1)$$

Show that thermal expansion coefficient $\alpha = 1/T$, independent of z . Note that in equation (1) we have used the fact that keeping the specific volume constant is equivalent to a constant local density. Calculate c_p and compare it with the result from part a).

- Calculate the heat capacity via the variance of \mathcal{H} and interpret the resulting terms.

Office Hours: Monday, October 26, 8-10 am (HIT K 43.2)